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Modelling the Differing Impacts of Covid-19 in the UK Labour Market

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Abstract

This paper models the impact of the Covid-19 pandemic on graduates and non-graduates in the UK. We use a model that is designed around key features of the UK labour market: (i) graduates typically earn higher wages and enjoy greater job security than non-graduates; (ii) many graduates are employed in non-graduate roles; and (iii) most newly hired workers are recruited from other jobs rather than from unemployment. We simulate this model using an array of shocks, designed to mimic the impact of the Covid-19 pandemic. In our baseline simulation, unemployment rises to 3 million workers, of whom 2.2 million are non-graduates. Employment falls by 6.6%, while non-graduate employment falls by 8.1%. The real wage falls by 0.9% while the wages of non-graduates fall by 2.0%. We find that the temporary Job Retention Scheme (JRS) has a large impact. Without the JRS, unemployment rises to 5 million, employment falls by 20% and real wages fall by 16%. But we also find that the JRS has increased the loss of output due to the pandemic. We find that a "jobs package", designed to reduce job destruction and increase job creation can reduce the rise in unemployment from 3 million to 2.2 million and accelerate the recovery in employment. Finally, we find that a "second wave" of infections that leads to a reinstatement of the lockdown in 2020Q4 would deepen and prolong the loss of output and increase unemployment to 4.5 million, an unemployment rate of 13.25%.

Keywords: Covid-19, Economic Impact, UK Labour Market; Search Frictions; Graduate Employment; DSGE model

JEL Classification: E23, E32, J23, J30, J64

1 Introduction

The Covid-19 pandemic has severely disrupted economies across the world and put health care systems under enormous stress. This paper models the response of the labour market in the UK to the pandemic. The Covid-19 crisis has had a major impact on the UK labour market. At the peak of the pandemic, only around 50% of workers were at work and up to 10 million workers were furloughed as part of the Job Retention Scheme (JRS) (Leslie (2020), PWC (2020)). Two thirds of employers made use of the JRS. GDP fell by by

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20% in April 2020, and is widely forecast to be at least 10% lower in 2020 compared to 2019. Unemployment is forecast to rise to at least 3 million, or 9% of the workforce[1].

We address four main questions. First, how has the pandemic affected the labour market experience of different types of workers in the UK and what is the likely adjustment process as the economy recovers? Second, what is the impact of the Job Retention Scheme, adopted by the Government in March 2020 as the core of their economic policy response to the pandemic? Third, what would be the impact of a "Jobs Package" that extends the duration of the JRS and introduces hiring subsidies in an attempt to accelerate job creation? And, fourth, what would be the effect of a "second wave" of infections in late 2020?

We address these questions using a DSGE model with labour market frictions, designed to reflect the UK labour market. We simulate this model using a series of shocks that are constructed to mimic the effects of the pandemic, and track the movements of output, employment, unemployment, job losses, wages and inflation over the course of the crisis. We adapt the models of Blanchard and Gali (2010), Ravenna and Walsh (2011), and Thomas (2008)[2] to allow for three key features of the UK labour market. First, the labour market experience of different types of worker differs markedly. Some workers, typically those with university-level qualifications, have higher wages and greater job security than non-graduates (ONS (2017*a*)). The pandemic has highlighted this disparity, as lower paid workers with lower qualifications were more likely to be in key occupations and at higher risk from the virus (Gustafsson and McCurdy (2020)). Second, the distribution of graduates across occupations is complex, with over one third of graduates employed in "non-graduate" occupations (ONS (2017*a*)). And third, there is substantial movement of workers between jobs and between sectors, with most hires coming from workers who are already employed rather than from the unemployed (ONS (2017*c*))[3].

Reflecting these features, our model distinguishes between workers with higher qualifications, who we identify as graduates, and workers with lower qualifications. We also distinguish between "high productivity" and "low productivity" firms; as we outline below, high productivity firms pay higher wages and offer greater job security. We assume that only workers with higher qualifications can work in high productivity firms, while all workers can work at low productivity firms[4] [5]. As we describe below, the UK labour market is characterised by flows of workers moving between high productivity jobs, moving between low productivity jobs, moving from low productivity jobs to high productivity jobs, and vice versa. There are also flows of workers from unemployment to and from high and low productivity jobs. In our model, we generate worker flows to match these data through job search. New hires into high productivity jobs can be from unemployment, from another high productivity job or from a low productivity job. New hires of lower qualification workers into low productivity jobs can come from unemployment or from another low productivity job. New hires of higher qualification workers into low productivity jobs can come from

[1]The Bank of England projects a 9% unemployment rate (BoE (2020*a*)). The Office for Budgetary Responsibility projects 11.9% (OBR (2020))

[2]Faccini et al. (2013) analyse this type of model for the UK.

[3]Only 28% of new hires in groups 1)-3) in the UK Standard Occupational Classification (SOC) (ONS (2009)): "Managers, directors & senior officials", "Professional occupations" and "Associate professional & technical", come from the unemployed. And only 46% of workers with lower qualifications into groups 4)-9) of the SOC come from the unemployed.

[4]Two sector DSGE models with labour market frictions have been developed to address policy issues in less developed countries, where the distinction between the "formal" and "informal" sectors is important (eg Castillo and Montoro (2012) and Mattesini and Rossi (2009).). In contrast to our model, these models typically assume that only unemployed workers can be hired. They also differ from our model by assuming either no mobility (so only high qualification workers can work at high productivity firms and only low qualification workers can work at low productivity firms) or complete mobility (all workers are identical and can do any job).

[5]Our model has similarities with the literature on "good jobs and bad jobs" (eg Gertler et al. (2020), and Faccini and Melosi (2020)). In these models, both employed and unemployed workers can be hired. The productivity of a job match is random and so low productivity job matches can arise even though workers and firms are identical. In our model, "bad matches" are not random; rather they arise when high qualification workers are employed by low productivity firms.

unemployment, from another low productivity job or from a high productivity job. In addition, since all jobs can end, there are also flows of high and low qualification workers out from low productivity firms into unemployment and flows of high qualification workers from high productivity firms into unemployment.

We model the impact of Covid-19 on the UK labour market as a series of simultaneous adverse shocks. Baqaee and Farhi (2020) use a similar approach, modelling the impact of the pandemic on the US as an "omnibus of various supply and demand shocks". Related work includes Maria del Rio-Chanona et al. (2020), who investgate how Covid-19 has led to the adverse supply and demand shocks in the US and Fornaro and Wolf (2020) and Guerrieri et al. (2020), who conceptualise the transmission of adverse supply shocks into adverse demand shocks. Other models (eg Eichenbaum et al. (2020)) incorporate simple Susceptible-Infectious-Recovered epidemiological processes into DSGE models, to analyse how households may have reduced labour supply and consumption demand in response to Covid-19, generating a large, persistent recession. Mihailov (2020) incorporates these effects into the DSGE model of Galí et al. (2020) to estimate the impact of Covid-19. Our model has similarities with this approach, but has a greater emphasis on the labour market experience of different types of workers, on labour market frictions and on wage bargaining.

We classify the shocks in our simulation as being either aggregate supply or aggregate demand shocks. Aggregate supply shocks comprise shocks that reduce the workforce due to workers being in self-isolation or sick with Covid-19; shocks that increase job destruction[6]; shocks that reduce productivity, due to employees working from home or being furloughed as part of the Job Retention Scheme[7]; and shocks to wages, due to the state paying a proportion of wages as part of the JRS. Reflecting UK evidence on the incidence of these shocks in different sectors, the severity of these shocks differs between high- and low productivity jobs. Aggregate demand shocks comprise shocks to aggregate demand that rise from households reducing expenditure in response to the pandemic, and having reduced opportunities to spend[8]; and shocks to the interest rate due to the monetary policy response to the crisis[9]. We also model a shock to the composition of aggregate demand to capture the especially large fall demand in the hospitality, leisure and related sectors[10].

Simulations of our model broadly match other predictions of the economic impact of the pandemic at the aggregate level. In our baseline scenario, output falls sharply but then recovers relatively quickly, returning to pre-pandemic levels by mid-2021. Unemployment increases to 3.0 million workers, an unemployment rate of 8.7%; unemployment recovers slowly and does not return to pre-pandemic levels until 2022. Our baseline scenario projects deflation in 2020, followed by several years of above-target inflation during the recovery from the pandemic. We find that real wages fall by 0.9%.

We find that the Covid-19 pandemic has exacerbated structural differences in the UK labour market as the pandemic has very different impacts on graduates compared to non-graduates. We project a large surge in job losses, an additional 1.2 million job losses for non-graduates and an additional 400,000 job losses for graduates[11]. As a result, the unemployment rate of non-graduates rises to 10.1% compared to 6.8%

[6]The impact of these shocks is modelled for the US by Arbex et al. (2020).

[7]Similar schemes are in place Germany, France, Switzerland, the Netherlands and other countries, based on the German "Kurzarbeit" model, (Rothwell and Van Drie (2020)).

[8]This has been partly offset by increases in Government expenditure; for example, expenditure on health and social care increased by nearly 50% in the early stages of the pandemic. These were financed by large increases in borrowing, around £50 billion per month.

[9]The Bank of England has responded strongly to the pandemic, announcing renewed Quantitative Easing purchases that increased the total stock of asset purchases to £745 billion by the end of June. It also undertook long-term lending to banks at low interest rates, with the intention that this results in increased low-rate credit flows to firms, and increased it's loan facility for the UK Treasury.

[10]Faria-e Castro (2020) models this effect using a shock to the marginal utility from consumption of the output of the service sector.

[11]This is consistent with evidence in Tomlinson (2020), who finds that job losses in 2020Q2 were concentrated in areas such as Hospitality, Retail and Construction, with high proportions of employment of lower qualification workers. This is partly offset

for graduates. The real wage of non-graduates falls more than the real wage of graduates. These effects unwind slowly over time; the structure of employment does not return to pre-pandemic levels until 2024. The more severe impact of the pandemic on non-graduates arises because the more difficult labour market environment for these workers is compounded by the differential impact of the pandemic, as sectors that were especially severely impacted by the crisis, such as tourism and hospitality, employ a larger proportion of lower qualification workers. We find that these two factors have roughly equal importance in explaining the impact of the pandemic on less highly qualified workers.

We find that the Job Retention Scheme has a major impact on labour market outcomes. To assess this, we consider a scenario in which the JRS was not implemented. In our scenario, we assume there is no wage subsidy and that the number of workers infected or in self isolation triples, since social distancing is more difficult at the workplace than at home. We make the conservative assumption that 25% of workers who were furloughed would have been made redundant in the absence of the scheme; this implies additional job losses of over 1.5 million workers, mainly non-graduates. We find that unemployment rises to 5.0 million or 14.8%, with an unemployment rate of 16.7% for non-graduates and 12.2% for graduates. Employment falls by 20%, with falls of 23% for non-graduates and 15% for graduates. The real wages of non-graduates and graduates fall by 24.0% and 11.1% respectively. Despite this, the fall in output is lower compared to the baseline. This is because worker productivity is higher in the absence of the JRS, since the negative impact of non-productive furloughed workers is removed.

We find that a "jobs package", designed to reduce job destruction and accelerate job creation, through a slower removal of the Job Retention Scheme and the introduction of subsidies to vacancy posting has a positive effect. Unemployment rises to 2.2 million, compared to 3 million in the baseline scenario. Unemployment of graduates and non-graduates are both around 400,000 lower. The rise in the unemployment rate peaks at 6.4%. The initial fall in employment is similar to the baseline, but the recovery in employment is accelerated by around 3 months. And the fall in real wages is avoided, as wages increase by 1.4%. We also simulate the impact of a "second wave" of infections that leads to another lockdown in 2020Q4. To do this, we assume that a second wave reinstates the values of the shocks of 2020Q2 in 2020Q4. Our results highlight the potential damage from a second wave. The loss of output is much larger and more prolonged than in the baseline. Unemployment rises to 4.5 million, an unemployment rate of 13.25%. Employment falls by 12% [12].

Research into an ongoing event of the scale and rarity of the Covid-19 pandemic must be treated with caution. In order to analyse the impact of the pandemic, this paper makes a series of strong assumptions. First, our model does not allow for uncertainty. Second, our model tracks 11 distinct labour market flows. This enables the model to capture some of the richness of the UK labour market, but also makes the model complex. This complexity comes at a cost, as our model does not analyse movements of workers in and out of self-employment or in and out of the labour force. Third, we assume that the pandemic does not affect the steady-state of the UK economy; and that the pandemic does not change structural relationships. Fourth, we assume it is appropriate to use a linearised version of the model[13], even though the pandemic moves the economy some distance away from the steady-state[14]. Further research that address the impact

by an increase in employment in the Health and Care sectors. By contrast, job losses in sectors with higher concentrations of more highly qualified workers, such as Finance and Insurance and Public Administration are 5-6 times lower.

[12] We do not model the possibility of other events, for example an adverse impact from Brexit or a severe second wave in other countries.

[13] Our use of a linearisation is pragmatic. Our model comprises over 60 relationships, many of which are nonlinear. Solving such a large nonlinear model is computationally impractical. Using a second-order expansion of such a large model is also impractical.

[14] Subjecting our model to the large scale shocks that are required to mimic the impact of the pandemic puts our simulations

of the pandemic using alternative approaches would be useful.

2 The Model

2.1 Overview

The economy is composed of households, wholesale firms, retail firms, the government and the Central Bank. Households are composed of two types of worker: those with high qualifications and those with low qualifications. There are two types of goods and two types of firms. High productivity wholesale firms use high qualification workers to produce high productivity wholesale goods. They sell these to high productivity retail firms who use them to produce high productivity retail goods, which they sell to households. Low productivity wholesale firms use high and low qualification workers to produce low productivity wholesale goods. They sell these to low productivity retail firms who use them to produce low productivity retail goods, which are sold to households. Wholesale goods markets are competitive but retail goods markets are imperfectly competitive. The government collects taxes and purchases retail goods. The Central Bank sets the interest rate on the financial asset, which households use to smooth consumption over time.

At the beginning of each period, workers search for jobs and vacancies are posted by wholesale firms without a productive job match. This results in new job matches being formed, which become productive in the same period. Next, wages are set and wholesale goods are produced. Then, job separation occurs. Some employer-worker job matches survive and continue into the next period; other matches break down. Separated workers enter the next period as unemployed and begin search for a new job then[15].

2.2 The Labour Market

There are two types of workers: high qualification workers L^{hq} and low qualification workers L^{lq}. In the pre-pandemic period $L^{hq} + L^{lq} = L$, where we use $L = 1$ as a normalisation. All high qualification workers are identical and all low qualification workers are identical. During the pandemic, the number of high qualification workers is $L_t^{hq} = L^{hq} e^{\varepsilon_t^{hq}}$ and the number of low qualification workers is $L_t^{lq} = L^{lq} e^{\varepsilon_t^{lq}}$, where ε_t^{hq} and ε_t^{lq} are shocks that capture the impact of the pandemic on the workforce.

High qualification workers can be employed by high or low productivity wholesale firms, but low qualification workers can only be employed by low productivity wholesale firms. In any period, u_t^{hq} high qualification workers are unemployed, $n_t^{l,hq}$ are employed by low productivity firms and $n_t^{h,hq}$ are employed by high productivity firms, so

$$L_t^{hq} = u_t^{hq} + n_t^{l,hq} + n_t^{h,hq} \tag{1}$$

Similarly, in any period, u_t^{lq} low qualification workers are unemployed and $n_t^{l,lq}$ are employed by low productivity firms, so

$$L_t^{lq} = u_t^{lq} + n_t^{l,lq} \tag{2}$$

Following production, job separation occurs, at rates τ_t^h and τ_t^l for high and low productivity firms respectively. We assume that $\tau_t^l = \tau^l e^{\varepsilon_t^{\tau^l}}$ and $\tau_t^h = \tau^h e^{\varepsilon_t^{\tau^h}}$, $\tau^l > \tau^h$, where $\varepsilon_t^{\tau^l}$ and $\varepsilon_t^{\tau^h}$ are shocks to the rate of

under stress; we find that values of vacancies and related labour market variables are highly volatile in the early phase of the pandemic, as firms first cease hiring in 2020Q2-Q3 and then seek to rebuild their workforce once recovery from the crisis begins.

[15] The Job Retention Scheme creates three choices for a firm: to continue a job match, to terminate it or to use a furlough. The option of a furlough adds an additional layer of complexity to the analysis of endogenous job destruction. This is beyond the scope of this paper, so we assume job destruction is exogenous

job destruction. The shocks will capture the wave of job losses induced by the pandemic.

We generate worker flows that match UK data through job search. All unemployed workers and all employed workers search for jobs. Search for a job offered by a high productivity firm comes from unemployed high qualification workers, high qualification workers employed by low productivity firms and from high qualification workers employed by another high productivity firm. Search for jobs at high productivity firms is

$$s_t^h = s_t^{h,hq,u} + s_t^{h,hq,l} + s_t^{h,hq,h} \tag{3}$$

$s_t^{h,hq,u} = \zeta^{h,hq,u} u_t^{hq}$ is search for high productivity jobs by unemployed high qualification workers, who each search with intensity $\zeta^{h,hq,u}$. $s_t^{h,hq,l} = \zeta^{h,hq,l} n_t^{l,hq}$ is search by high qualification workers employed by low productivity firms, who each search for jobs at high productivity firms with intensity $\zeta^{h,hq,l}$. And $s_t^{h,hq,h} = \zeta^{h,hq,h} n_t^{h,hq}$ is search by high qualification workers employed by high productivity firms, who each search with intensity $\zeta^{h,hq,h}$.

Search for a job offered by a low productivity firm comes from unemployed low and high qualification workers, from low qualification workers employed by another low productivity firm, from high qualification workers employed by another low productivity firm, and from high qualification workers employed by high productivity firms. So search for jobs at low productivity firms is

$$s_t^l = s_t^{l,lq,u} + s_t^{l,lq,l} + s_t^{l,hq,u} + s_t^{l,hq,h} + s_t^{l,hq,l} \tag{4}$$

$s_t^{l,lq,u} = \zeta^{l,lq,u} u_t^{lq}$ and $s_t^{l,hq,u} = \zeta^{l,hq,u} u_t^{hq}$ are search for low productivity jobs by unemployed low and high qualification workers respectively, who search with intensity $\zeta^{l,lq,u}$ and $\zeta^{l,hq,u}$. $s_t^{l,lq,l} = \zeta^{l,lq,l} n_t^{l,lq}$, $s_t^{l,hq,h} = \zeta^{l,hq,h} n_t^{h,hq}$, and $s_t^{l,hq,l} = \zeta^{l,hq,l} n_t^{l,hq}$ are search by low qualification workers employed by low productivity firms, search by high qualification workers employed by high productivity firms and search by high qualification workers employed by low productivity firms, who search for jobs with intensity $\zeta^{l,lq,l}$, $\zeta^{l,hq,h}$ and $\zeta^{l,hq,l}$ respectively. We assume that workers search with fixed intensity. This is in line with most of the recent literature, including Moscarini and Postel-Vinay (2017), Morcarini and Postel-Vinay (2018) and Faccini and Melosi (2020), who model search by employed and unemployed workers in a one-sector model with identical firms and workers. Leduc and Liu (2020) model variable search intensity in a one-sector model where only unemployed workers can search for jobs and Gertler et al. (2020) analyse a one-sector model in which the productivity of a job match is randomly assigned to be high or low. The unemployed and workers in high productivity jobs search with a fixed intensity, but workers looking to move up from a low productivity match to a high productivity match search with varying intensity. In total, we use labour market search to generate 11 distinct types of worker flows in our two-sector model. This complexity makes consideration of variable search intensity impractical.

High productivity wholesale firms post vacancies v_t^h. Only high qualification workers can be matched to vacancies posted by high productivity firms. High productivity job matches are formed with the matching function

$$h_t^h = m^h (v_t^h)^{\alpha_h} (s_t^h)^{1-\alpha_h} \tag{5}$$

Job matches are formed with unemployed high qualification workers, high qualification workers currently employed in low and high productivity firms, so $h_t^h = h_t^{h,hq,u} + h_t^{h,hq,l} + h_t^{h,hq,h}$, where $h_t^{h,hq,u}$ is the number of unemployed high qualification workers hired by high productivity firms, $h_t^{h,hq,l}$ is the number of high qualification workers employed in low productivity firms who find a new job at a high productivity firm and

$h_t^{h,hq,h}$ is the number of high qualification workers employed in other high productivity firms who find a new job at a high productivity firm. We assume that the proportion of hires from each group depends on their relative search, so $h_t^{h,hq,u} = \frac{s_t^{h,hq,u}}{s_t^h} h_t^h$, $h_t^{h,hq,l} = \frac{s_t^{h,hq,l}}{s_t^h} h_t^h$, and $h_t^{h,hq,h} = \frac{s_t^{h,hq,h}}{s_t^h} h_t^h$.

Labour market tightness for high productivity firms is

$$\theta_t^h = \frac{v_t^h}{s_t^h} \tag{6}$$

so high productivity firms fill their vacancies at rate $q_t^h = \frac{h_t^h}{v_t^h}$ and the rate at which high qualification workers are matched with a vacancy at a high productivity firm, per unit of search, is $f_t^h = \frac{h_t^h}{s_t^h}$. The job finding rate of unemployed high qualification workers seeking work in high productivity firms is[16] $f_t^{h,hq,u} = \frac{h_t^{h,hq,u}}{u_t^{hq}} = \zeta^{h,hq,u} f_t^h$. Similarly, the job finding rates of high qualification workers currently employed in low or high productivity firms seeking work in high productivity firms are $f_t^{h,hq,l} = \frac{h_t^{h,hq,l}}{n_t^{l,hq}} = \zeta^{h,hq,l} f_t^h$ and $f_t^{h,hq,h} = \frac{h_t^{h,hq,h}}{n_t^{h,hq}} = \zeta^{h,hq,h} f_t^h$ respectively.

Low productivity wholesale firms post vacancies v_t^l. Low productivity job matches are formed with the matching function

$$h_t^l = m^l (v_t^l)^{\alpha_l} (s_t^l)^{1-\alpha_l} \tag{7}$$

We again assume that the proportion of hires from each source of hires depends on their relative search. The number of unemployed low qualification workers hired by low productivity firms is $h_t^{l,lq,u} = \frac{s_t^{l,lq,u}}{s_t^l} h_t^l$ and the number of unemployed high qualification workers hired by low productivity firms is $h_t^{l,hq,u} = \frac{s_t^{l,hq,u}}{s_t^l} h_t^l$. The number of low qualification workers employed in low productivity jobs who find jobs at other low productivity firms is $h_t^{l,lq,l} = \frac{s_t^{l,lq,l}}{s_t^l} h_t^l$, and the number of high qualification workers employed by high productivity firms who are hired by low productivity firms is $h_t^{l,hq,h} = \frac{s_t^{l,hq,h}}{s_t^l} h_t^l$. Low productivity market tightness is

$$\theta_t^l = \frac{v_t^l}{s_t^l} \tag{8}$$

so low productivity firms fill their vacancies at rate $q_t^l = \frac{h_t^l}{v_t^l}$ and the rate at which workers are matched with a vacancy at a low productivity firm is $f_t^l = \frac{m_t^l}{s_t^l}$. The job finding rates of unemployed low and high qualification workers seeking work in low productivity firms are $f_t^{l,lq,u} = \frac{h_t^{l,lq,u}}{u_t^{lq}} = \zeta^{l,lq,u} f_t^l$ and $f_t^{l,hq,u} = \frac{h_t^{l,hq,u}}{u_t^{hq}} = \zeta^{l,hq,u} f_t^l$ respectively. And the job finding rates of low qualification workers employed in low productivity firms and high qualification workers employed in high productivity firms seeking work in low productivity firms are $f_t^{l,lq,l} = \frac{h_t^{l,lq,l}}{n_t^{l,lq}} = \zeta^{l,lq,l} f_t^l$ and $f_t^{l,hq,h} = \frac{h_t^{l,hq,h}}{n_t^{h,hq}} = \zeta^{l,hq,h} f_t^l$ respectively.

[16]Since $f_t^{h,hq,u} = \frac{h_t^{h,hq,u}}{u_t^{hq}} = \frac{s_t^{h,hq,u}}{s_t^h} \frac{h_t^h}{u_t^{hq}} = \frac{s_t^{h,hq,u}}{u_t^{hq}} f_t^h = \zeta^{h,hq,u} f_t^h$

2.3 Households

Household members collectively derive utility from consumption. The household utility function is

$$H_t = E_t \sum_{k=0}^{\infty} \beta e^{\varepsilon^d_{t+k}} \frac{C_{t+k}^{1-\eta}}{1-\eta} \tag{9}$$

where C is consumption and e^{ε^d} is a preference shock. We assume that $\varepsilon^d < 0$ during the pandemic and its aftermath.

As discussed below, all high qualification workers in high productivity jobs earn the same real wage, w_t^h and all workers, high qualification or low qualification, in low productivity jobs earn the same real wage, w_t^l. The budget constraint of the household is

$$P_t w_t^l n_t^l + P_t w_t^h n_t^{h,hq} + P_t b u_t + B_{t-1} + \Pi_t - T_t = P_t C_t + q_t B_t \tag{10}$$

where P is the consumption price index, $u_t = u_t^{lq} + u_t^{hq}$ is the number of unemployed workers, $n_t^l = n_t^{l,hq} + n_t^{l,lq}$ is the number of workers employed by low productivity firms, b is the real opportunity cost of employment, comprising the value of leisure and unemployment benefit, $q = \frac{1}{1+i}$ is the nominal price of bonds, Π is the profit the household receives for the ownership of firms and T_t is a lump-sum tax levied on the household by the government.

The household chooses consumption and bond purchases to maximise utility subject to their budget constraint. The optimality condition for consumption and bonds gives the Euler equation

$$C_t^{-\eta} = \beta e^{\varepsilon_t^d} E_t C_{t+1}^{-\eta} \frac{1+i_t}{1+E_t \pi_{t+1}} \tag{11}$$

The real interest rate $r_t = \frac{1+i_t}{1+E_t\pi_{t+1}}$, so equation (11) implies that the stochastic discount factor is

$$E_t \beta_{t,t+1} = \beta e^{\varepsilon_t^d} \frac{E_t C_{t+1}^{-\eta}}{C_t^{-\eta}} \tag{12}$$

The household derives utility from consuming both high productivity retail goods and low productivity retail goods. We assume

$$C_t = \left[(\Gamma_t^h)^{\frac{1}{\nu}} (C_t^h)^{\frac{\nu-1}{\nu}} + (1-\Gamma_t^h)^{\frac{1}{\nu}} (C_t^l)^{\frac{\nu-1}{\nu}} \right]^{\frac{1}{\nu-1}} \tag{13}$$

where C^h is consumption of high productivity retail goods, C^l is consumption of low productivity retail goods and ν is the elasticity of substitution between them. Γ_t^h is the proportion of household consumption that is of high productivity retail goods and $\Gamma_t^h = \Gamma_h e^{\varepsilon^{\Gamma_t^h}}$, where $\varepsilon^{\Gamma_t^h}$ is a shock to the preference for high productivity retail goods relative to low productivity retail goods. We use this shock in modelling the impact of the pandemic on the demand for different types of goods. The implied price index is

$$P_t = \left[\Gamma_t^h (P_t^h)^{1-\nu} + (1-\Gamma_t^h)(P_t^l)^{1-\nu} \right]^{\frac{1}{1-\nu}} \tag{14}$$

where P_t^h is the price index for high productivity retail goods and P_t^l is the price index for low productivity retail goods. The demand for high productivity and low productivity retail goods is

$$C_t^h = \Gamma_t^h \Big(\frac{P_t^h}{P_t}\Big)^{-\nu} C_t \tag{15}$$

and

$$C_t^l = (1-\Gamma_t^h)\Big(\frac{P_t^l}{P_t}\Big)^{-\nu} C_t \tag{16}$$

Household consumption of high productivity retail goods is a composite of individual high productivity retail goods defined by $C_t^h = (\int_0^1 (C_{jt}^h)^{\frac{\nu^h-1}{\nu^h}} dj)^{\frac{\nu^h}{\nu^h-1}}$, where C_j^h is household consumption of high productivity retail good j. The price index for high productivity retail goods is $P_t^h = (\int_0^1 (P_{jt}^h)^{(1-\nu^h)} dj)^{\frac{1}{1-\nu^h}}$ where P_j^h is the price of high productivity retail good j.

Similarly, household consumption of low productivity retail goods is a composite of individual low productivity retail goods defined by $C_t^l = (\int_0^1 (C_{jt}^l)^{\frac{\nu^l-1}{\nu^l}} dj)^{\frac{\nu^l}{\nu^l-1}}$, where C_j^l is household consumption of low productivity retail good j. The corresponding price index is $P_t^l = (\int_0^1 (P_{jt}^l)^{(1-\nu^l)} dj)^{\frac{1}{1-\nu^l}}$ where P_j^l is the price of low productivity retail good j.

Households purchase high productivity retail good j from the retail firm in the high productivity retail sector that sells this good. Household demand is

$$C_{jt}^h = \Big(\frac{P_{jt}^h}{P_t^h}\Big)^{-\nu^h} C_t^h \tag{17}$$

Similarly, households purchase low productivity retail good j from the retail firm in the low productivity retail sector that sells this good. Household demand for this good is

$$C_{jt}^l = \Big(\frac{P_{jt}^l}{P_t^l}\Big)^{-\nu^l} C_t^l \tag{18}$$

2.4 The Government, the Central Bank and Aggregate Demand

Aggregate demand is the sum of demand from households and the Government[17]

$$Y_t = C_t + G_t \tag{19}$$

We assume $G_t = Ge^{\varepsilon_t^g}$, where ε^g is a government expenditure shock. We assume $\varepsilon^g > 0$ during the pandemic and its aftermath. Government demand for output is the sum of the demand for high and low productivity retail goods, so

$$G_t = G_t^h + G_t^l \tag{20}$$

We assume that government expenditure does not distort the pattern of aggregate demand, so $\frac{G_t^h}{G_t} = \frac{C_t^h}{C_t}$.

We assume that the Central Bank sets the interest rate using the simple Taylor rule

$$i_t = \bar{i} + \phi_\pi \pi_t + \phi_y \hat{y}_t + \varepsilon_t^i \tag{21}$$

where $\hat{y}$ is the output gap and ε_t^i is a monetary policy shock.

[17]An alternative formulation of this $Y_t = C_t + G_t + \gamma^h v_t^h + \gamma^l v_t^l$, where γ^h and γ^l are the costs of posting a vacancy for high and low productivity firms, respectively. In our simulations, the increased volatility of vacancies in 2020Q2 and 202Q3 distorts this relationship, leading to unreliable results.

2.5 Wholesale Firms

2.5.1 High Productivity Wholesale Firms

All high productivity wholesale firms are competitive and identical. There is no price rigidity in wholesale prices, so all high productivity wholesale firms set the same price. The objective function of the high productivity wholesale firm is

$$J_t^h = E_t \sum_{k=0}^{\infty} \frac{\beta^{t+k}\Lambda_{t+k}}{\Lambda_t} \Big\{ \frac{P_t^{h,W}}{P_t^h} Y_{t+k}^h - w_{t+k}^h n_{t+k}^h - \gamma^h v_{t+k}^h \Big\} \tag{22}$$

where Y^h is output, $P^{h,W}$ is the price of the output of high productivity wholesale firms, P^h is the price of the output of high productivity retail firms and γ^h is the cost of posting a vacancy for high productivity firms. The production function is

$$Y_t^h = A_t^h n_t^{h,hq} \tag{23}$$

where $A_t^h = A^h e^{\varepsilon_t^{s^h}}$, where $\varepsilon_t^{s^h}$ is a shock to the productivity of workers at high productivity firms. $\varepsilon_t^{s^h} < 0$ during the pandemic, as some employed workers will be furloughed under the Job Retention Scheme and others will be working from home, where they are less productive. Considering the evolution of employment, $n_t^{h,hq}$ high qualification workers are employed and used in production in period t. Following production, $\tau_t^h n_t^{h,hq}$ workers are separated and become unemployed. At the start of period $t+1$, $f_{t+1}^{h,hq,h} n_t^{h,hq}$ workers move to other high productivity firms and $f_{t+1}^{l,hq,h} n_t^{h,hq}$ move to a low productivity firm. The firm posts v_{t+1}^h vacancies and recruits $q_{t+1}^h v_{t+1}^h$ new high qualification workers. Defining $\rho_t^h = 1 - \tau_t^h - f_{t+1}^{h,hq,h} - f_{t+1}^{l,hq,h}$, the evolution of employment for the high productivity wholesale firm is therefore

$$n_{t+1}^h = \rho_t^h n_t^h + q_{t+1}^h v_{t+1}^h \tag{24}$$

The firm chooses the number of vacancies to post to maximise (22) subject to (23) and (24). The optimality condition is

$$\frac{\partial J_{t+1}^h}{\partial v_{t+1}^h} = -\gamma^h + q_{t+1}^h E_t \frac{\partial J_{t+1}^h}{\partial n_{t+1}^h} = 0 \tag{25}$$

where $\frac{\partial J_t^h}{\partial n_t^h} = \frac{A_t^h}{\mu^h} - w_t^h + E_t \rho_t^h \beta_{t,t+1} \frac{\partial J_{t+1}^h}{\partial n_{t+1}^h}$ and where $\mu^h = \frac{P_t^h}{P_t^{h,W}}$. Noting that (25) implies $\frac{\partial J_{t+1}^h}{\partial n_{t+1}^h} = \frac{\gamma^h}{E_t q_{t+1}^h}$, and so $\frac{\partial J_t^h}{\partial n_t^h} = \frac{A_t^h}{\mu^h} - w_t^h + E_t \rho_t^h \beta_{t,t+1} \frac{\gamma}{q_{t+1}^h}$, the optimality condition implies

$$\frac{A_t^h}{\mu^h} = w_t^h + \lambda_t^h \tag{26}$$

where $\lambda_t^h = \gamma^h \big(\frac{1}{q_t^h} - E_t \rho_t^h \beta_{t,t+1} \frac{1}{q_{t+1}^h}\big)$

2.5.2 Low Productivity Wholesale Firms

Low productivity wholesale firms employ both high qualification and low qualification workers, so employment is $n_t^l = n_t^{l,hq} + n_t^{l,lq}$. Due to a legal or fairness constraint, all workers employed at the firm must be paid the same wage w_t^l. The objective function of low productivity wholesale firms is

$$J_t^l = \sum_{k=0}^{\infty} \beta^{t+k} \frac{\Lambda_{t+k}}{\Lambda_t} \Big\{ \frac{P_t^{l,W}}{P_t^l} Y_{t+k}^l - w_{t+k}^l n_{t+k}^l - \gamma^l v_{t+k}^l \Big\} \tag{27}$$

where Y^l is output, $P^{l,W}$ is the price of the output of low productivity wholesale firms, P^l is the price of the output of low productivity retail firms and γ^l is the cost of posting a vacancy for low productivity firms. The production function is

$$Y_t^l = A_t^l n_t^l \tag{28}$$

where $A_t^l = A^l e^{\varepsilon_t^{s^l}}$, where $\varepsilon_t^{s^l}$ is a shock to the productivity of workers at low productivity firms; we assume that high qualification and low qualification workers are equally productive in the low productivity employment. $\varepsilon_t^{s^l} < 0$ during the pandemic, due to some employed workers being furloughed and others working from home.

The evolution of employment at low productivity firms reflect the different evolutions of employment of high qualification and low qualification workers. Considering high qualification employment, $n_t^{l,hq}$ workers are employed and used in production in period t. Following this, $\tau_t^l n_t^{l,hq}$ workers are separated. In period $t+1$, $f_{t+1}^{h,hq,l} n_t^{l,hq}$ workers move to high productivity firms and $f_{t+1}^{l,hq,l} n_t^{l,hq}$ workers move to other low productivity firms. The firm posts v_{t+1}^l vacancies and recruits $q_{t+1}^{l,hq} v_{t+1}^l$ new high qualification workers. Defining $\rho_t^{l,hq} = 1 - \tau_t^l - f_{t+1}^{h,hq,l} - f_{t+1}^{l,hq,l}$, the evolution of high qualification employment at the representative low productivity wholesale firm is therefore

$$n_{t+1}^{l,hq} = \rho_t^{l,hq} n_t^{l,hq} + q_{t+1}^{l,hq} v_{t+1}^l \tag{29}$$

Following similar arguments, the evolution of low qualification employment at the representative low productivity wholesale firm is

$$n_{t+1}^{l,lq} = \rho_t^{l,lq} n_t^{l,lq} + q_{t+1}^{l,lq} v_{t+1}^l \tag{30}$$

where $\rho_t^{l,lq} = 1 - \tau_t^l - f_{t+1}^{l,lq,l}$. The evolution of employment at the firm is therefore

$$n_{t+1}^l = \left\{ \rho_t^{l,hq} n_t^{l,hq} + \rho_t^{l,lq} n_t^{l,lq} \right\} + (q_t^{l,hq} + q_t^{l,lq}) v_t^l \tag{31}$$

Defining $\delta_{jt}^{l,hq} = \frac{n_t^{l,hq}}{n_t^l}$ as the proportion of the firm's workforce that are high qualification and $q_t^l = q_t^{l,hq} + q_t^{l,lq}$, we obtain

$$n_{t+1}^l = \rho_t^l n_t^l + q_t^l v_t^l \tag{32}$$

where $\rho_t^l = \rho_t^{l,hq} \delta_t^{l,hq} + \rho_t^{l,lq}(1 - \delta_t^{l,hq})$.

The firm chooses the number of vacancies to post to maximise(27) subject to (28) and (32). The optimality condition is

$$\frac{\partial J_{t+1}^l}{\partial v_{t+1}^l} = -\gamma^l + q_{t+1}^l E_t \frac{\partial J_{t+1}^l}{\partial n_{t+1}^l} = 0 \tag{33}$$

where $\frac{\partial J_t^l}{\partial n_t^l} = \frac{A_t^l}{\mu^l} - w_t^l + E_t \rho_t^l \beta_{t,t+1} \frac{\partial J_{t+1}^l}{\partial n_{t+1}^l}$ and $\mu^l = \frac{P_t^l}{P_t^{l,W}}$. Noting that (33) implies $\frac{\partial J_{t+1}^l}{\partial n_{t+1}^l} = \frac{\gamma^l}{E_t q_{t+1}^l}$, and so $\frac{\partial J_t^l}{\partial n_t^l} = \frac{A_t^l}{\mu^l} - w_t^l + E_t \rho_t^l \beta_{t,t+1} \frac{\gamma}{q_{t+1}^l}$, the optimality condition implies

$$\frac{A_t^l}{\mu^l} = w_t^l + \lambda_t^l \tag{34}$$

where $\lambda_t^l = \gamma^l \left(\frac{1}{q_t^l} - E_t \rho_t^l \beta_{t,t+1} \frac{1}{q_{t+1}^l}\right)$

2.6 Wage Determination

In this model, wages are determined through wage bargaining, with real wage rigidity, where we model real wage rigidity following Krause and Lubik (2007) and Faia (2008). We also model the impact of a temporary wage subsidy due to the Job Retention Scheme, which we treat as a shock. The wage paid by high productivity firms is

$$w_t^h = \{\varphi^h w^h + (1-\varphi^h) w_t^{b,h}\} e^{-\varepsilon_t^{w^h}} \tag{35}$$

where $w_t^{b,h}$ is the wage implied by bargaining, w^h is the steady-state value of this wage, φ^h captures real wage rigidity and ε^{w^h} is the wage subsidy. The wage received by workers employed in high productivity firms is

$$w_t^{h,w} = \varphi^h w^h + (1-\varphi^h) w_t^{b,h} \tag{36}$$

The bargained wage is chosen to maximise

$$S_t = (S_t^{h,hq})^{\zeta^h} (F_t^h)^{1-\zeta^h} \tag{37}$$

where $S_t^{h,hq}$ is the surplus to the household from an additional worker being employed in a high productivity firm, F_t^h is the surplus to the firm and ζ^h is the bargaining power of high qualfication workers in high productivity jobs. This gives the sharing rule

$$(1-\zeta^h) S_t^{h,hq} = \zeta^h F_t^h \tag{38}$$

As we show in the Appendix, this results in

$$w_t^{b,h} = \zeta^h \left\{ \frac{P_t^{h,W}}{P_t^h} A_t^h + \gamma^h \zeta^{h,hq,u} E_t \theta_{t+1}^h + \gamma^l \zeta^{l,hq,u} E_t \theta_{t+1}^l \right\} + (1-\zeta^h) b \tag{39}$$

so the wage for high productivity wholesale firms is determined by (35) and (39).

Using a similar notation, the wage paid by low productivity firms is

$$w_t^l = \{\varphi^l w^l + (1-\varphi^l) w_t^{b,l}\} e^{-\varepsilon_t^{w^l}} \tag{40}$$

and the wage received by workers employed in low productivity firms is

$$w_t^{l,w} = \varphi^l w^l + (1-\varphi^l) w_t^{b,l} \tag{41}$$

We assume that wage bargaining takes place between the firm and low qualification workers[18]. The bargained wage is determined by the sharing rule

$$(1-\zeta^l) S_t^{l,lq} = \zeta^l F_t^l \tag{42}$$

where $S_t^{l,lq}$ is the surplus to the household from an additional low qualification worker being employed in a low productivity firm, F_t^l is the surplus to the firm and ζ^l is the bargaining power of low qualification

[18] Although high qualification and low qualification workers have the same productivity and must be paid the same wage, a match with a low qualification worker has a different value to a low productivity firm than a match with a high qualification worker, because the respective matches break down with different probabilities.

workers in the low productivity sector. This implies (see Appendix for details)

$$w_t^{b,l} = \zeta^l \Big\{ \frac{P_t^{l,W}}{P_t^l} A_t^l + \gamma^l \zeta^{l,lq,u} E_t \beta_{t,t+1} \theta_{t+1}^l \Big\} + (1-\zeta^l) b \tag{43}$$

2.7 Retail Firms

2.7.1 High Productivity Retail Firms

High productivity retail firms produce differentiated high productivity retail goods, which they sell to households. High productivity retail firms face a downward sloping demand curve and determine the price of their output, acting as monopolistic competitors. High productivity retail firms purchase high productivity wholesale goods in a competitive market and transform these costlessly into a differentiated high productivity retail good.

The production function for the high productivity retail firm is

$$Y_t^h = Y_t^{h,W} \tag{44}$$

where $Y^{h,W}$ is the amount of high productivity wholesale goods purchased by the high productivity retail firm. High productivity sector retail firms can adjust their price in each period with probability $(1-\omega^h)$. In period t, they therefore choose their price, P_t^h, to maximise

$$E_t \sum_{k=0}^{\infty} (\omega^h)^k \Big\{ \beta_{t,t+k} \Big(\frac{P_t^h - P_{t+k}^{h,W}}{P_{t+k}^h} \Big) Y_{t+k}^h \Big\} \tag{45}$$

subject to

$$Y_t^h = \Big(\frac{P_t^h}{P_t^h} \Big)^{-\nu^h} Y_t^h \tag{46}$$

This gives the optimal price of the high productivity firm's finished good

$$\frac{P_t^{*h}}{P_t^h} = \mu(1-\beta\omega^h) E_t \sum_{k=0}^{\infty} (\omega^h)^k \beta_{t,t+k} mc_{t+k}^h \tag{47}$$

where $mc_{t+k}^h = \frac{P_t^{h,W}}{P_{t+k}^h}$ is the price of the high productivity intermediate good and $\mu^h = \frac{\nu^h}{\nu^h - 1}$ is the markup, high productivity retail firms set their price as a markup on their marginal cost.

High productivity retail firms that are unable to reset their price maintain their past optimal price indexed for inflation. This is consistent with the price setting rule

$$P_t^h = (1-\omega^h) P_t^{*h} + \omega^h \pi_{t-1}^h P_{t-1}^h \tag{48}$$

Taking the log deviation of (47) and the price setting rule around a zero inflation steady state gives the Phillips Curve for the high productivity sector

$$\pi_t^h = \frac{1}{1+\beta} \pi_{t-1}^h + \kappa^h \widehat{mc}_t^h + \frac{\beta}{1+\beta} \pi_{t+1}^h \tag{49}$$

where $\kappa^h = \frac{(1-\omega^h)(1-\beta\omega^h)}{\omega^h}$ is the slope.

2.7.2 Low productivity Retail Firms

Similarly, each low productivity retail firm produces a differentiated low productivity retail good, which it sells to households. Each low productivity retail firm faces a downward sloping demand curve and determines the price of their output, acting as a monopolistic competitor. Each low productivity retail firm purchases wholesale low productivity goods in a competitive market and transforms these costlessly into a differentiated low productivity retail good. Following the same argument as for high productivity retail firms, we obtain the price setting rule

$$P_t^l = (1-\omega^l)P_t^{*l} + \omega^l \pi_{t-1}^l P_{t-1}^l \tag{50}$$

where

$$\frac{P_t^{*l}}{P_t^l} = \mu(1-\omega^l\beta)E_t\sum_{k=0}^{\infty}(\omega^l)^k\beta_{t,t+k}mc_{t+k}^l \tag{51}$$

$mc_{t+k}^l = \frac{P_t^{l,W}}{P_{t+k}^l}$ is the price of the low productivity intermediate good and $\mu^l = \frac{\nu^l}{\nu^l-1}$. Taking the log deviation of (51) and (50) around a zero steady state gives the Phillips Curve

$$\pi_t^l = \frac{1}{1+\beta}\pi_{t-1}^l + \kappa^l\widehat{mc}_t^l + \frac{\beta}{1+\beta}\pi_{t+1}^l \tag{52}$$

where $\kappa^l = \frac{(1-\omega^l)(1-\beta\omega^l)}{\omega^l}$ is the slope.

3 Calibration

There are no similar studies of the UK labour market that distinguish between different types of workers. We therefore first construct a series of calibration targets. We define a high qualification worker as being educated to degree level or higher. The most recent data on graduates in the UK labour market is for 2017 (ONS (2017*a*)). We assume that the UK labour market was in steady-state, relative to the impact of the pandemic, in that year. In 2017, 42% of the population aged 21-64 were graduates; so we set $L^{hq} = 0.42$ and hence $L^{lq} = 0.58$. The unemployment rate of graduates in that year was 3%, so $\frac{u^{hq}}{L^{hq}} = 0.03$. this implies that $u^{hq} = \frac{u^{hq}}{L^{hq}}\frac{L^{hq}}{L^t} = 0.013$. Since the workforce is normalised to 1, the unemployment rate is $u = u^{hq} + u^{lq}$. The unemployment rate in 2017 was 4.3%, so $u^{lq} = 0.043 - 0.013 = 0.030$. Employment of graduates was $n^{hq} = L^{hq} - u^{hq} = 0.407$. In 2017, 36.3% of employed graduates were in non-graduate occupations (ONS (2017*a*)). So we use $n^{h,hq} = 0.407*(1-0.363) = 0.26$ and $n^{l,hq} = 0.407*0.363 = 0.15$ as calibration targets. Employment of non-graduates was $n^{l,lq} = L^{lq} - u^{lq} = 0.55$; we also target this value.

We target the wage in high productivity occupations relative to the wage in low productivity occupations. To construct this, we follow ONS practice and define a high productivity job as corresponding to groups 1)-3) in the UK Standard Occupational Classification (SOC) (ONS (2009)): "Managers, directors & senior officials", "Professional occupations" and "Associate professional & technical". We assume that high productivity firms only employ workers in these occupations and that only graduates are able to occupy these roles. We define a low productivity job as corresponding to groups 4)-9)[19]: "Administrative & secretarial", "Skilled trades", "Caring, leisure & other services", "Sales & customer services, "Process, plant & machine operatives" and "Elementary occupations". We assume that low productivity firms only employ workers in these occupations and that all workers are able to occupy these roles. Using ONS data for 2017 on employ-

[19]The ONS classifies groups 4-6 as medium skill and groups 7-9 as low skill

ment by occupation (ONS (2017*a*)) and weekly earnings by occupation (ONS (2017*b*)), we calculated that the average wage in high productivity occupations exceeds that in low productivity occupations by 190%. We target this in our calibrations.

For the rate of job destruction, we use data from the 2017 UK Labour Force Survey. This gives annual rates of job destruction for different occupations. We match these occupations to the UK SOC and use this to construct job survival rates for high and low productivity occupations. We obtain $\rho^h = 0.92$ and $\rho^l = 0.88$ as annual rates. In our calibration, we assume that a time period corresponds to one quarter. We use $\rho^h = 0.98$ and $\rho^l = 0.97$ as our quarterly calibration targets.

We construct calibration targets for job flows using data on flows of workers between employment and unemployment and between employment in different occupations. In steady-state, hires of high qualification workers into the high productivity sector are $h_{h,hq} = \rho^h n^{h,hq} = 0.005$. These hires can come from workers in the high or low productivity sectors, or from unemployment. To calculate these flows, we used ONS data on job to job moves by skill level in 2017 (ONS (2017*c*)) to construct measures of job movements within the high productivity sector and of hires into the high productivity sector from the low productivity sector. We also combined data on total hires from unemployment (ONS (2020*c*)) with estimates of the percentage of hires from unemployment that went to the high productivity sector (ONS (2016))[20] to construct a measure of hires into the high productivity sector from unemployment. This gave the relative sizes of the various flows into the high productivity sector: 28% of hires by high productivity firms came from unemployed high qualification workers, 55% were from workers employed at other high productivity firms and 17% were hired from low productivity firms.

Using a similar approach, the number of high qualification workers hired into the low productivity sector is $h^{l,hq} = \rho^l n^{l,hq} = 0.004$. These hires can come from workers in the high or low productivity sectors, or from unemployment. We obtain the number hired from the high productivity sector using ONS job-to-job flow data (ONS (2017*c*)). To find the number hired from the low productivity sector, we obtained the share of high qualification workers in the low productivity sector as $\frac{n^{l,hq}}{n^{l,hq}+n^{l,lq}} = 0.211$ and assumed that the share of high qualification workers in hires into the low productivity sector was equal to this, so $\frac{h^{l,hq,l}}{h^{l,hq,l}+h^{l,lq,l}} = 0.211$. We combined this with data on job moves within the low productivity sector to obtain hires of high qualification workers in the low productivity sector from elsewhere in that sector. We obtained estimates of hires of high qualification workers into the low productivity sector from unemployment by combining our estimate of hires into the low productivity sector from unemployment with the assumption that the share on high qualification workers in these hires matched their share in low productivity employment. Using all these data, we find that 27% of hires of high qualification workers by low productivity firms came from unemployment, 43% were from workers employed at high productivity firms and 31% were hired from other low productivity firms. Using a similar approach to estimate hires of low qualification workers into the low productivity sector, we find the number of low qualification workers hired into the low productivity sector is $h^{l,lq} = \rho^l n^{l,lq} = 0.016$. Of these, 46% of hires of low qualification workers by low productivity firms came from unemployment and 54% were hired from other low productivity firms. This evidence shows that only a minority of new hires come from the unemployed, highlighting the importance of modelling job-to-job flows in the UK labour market.

This gives a total of 20 calibration targets, outlined in Table 1).

Table 1) Calibration Targets

[20] This measure should be treated with caution as it is for London only

Steady-state Value	Interpretation	Target	Source	This Paper
u^{hq}	Graduate Unemployment Rate	0.013	ONS data	0.018
u^{lq}	Non-Graduate Unemployment Rate	0.030	ONS data	0.030
$n^{h,hq}$	Emp of Graduates in High Productivity Firms	0.26	ONS data	0.26
$n^{l,hq}$	Emp of Graduates in Low Productivity Firms	0.15	ONS data	0.15
$n^{l,lq}$	Emp of non-Graduates in Low Productivity Firms	0.55	ONS data	0.56
$\frac{w^h}{w^l}$	Relative Wage	1.90	ONS data	1.98
ρ^h	Match Continuation in High Productivity Firms	0.98	LFS data	0.99
ρ^l	Match Continuation in Low Productivity Firms	0.97	LFS data	0.98
$h^{h,hq}$	Hires of Graduates Into High Productivity Firms	0.005	authors calculation	0.003
$h^{l,hq}$	Hires of Graduates Into Low Productivity Firms	0.004	authors calculation	0.003
$h^{l,lq}$	Hires of Non-Graduates Into Low Productivity Firms	0.016	authors calculation	0.015
$\frac{h^{h,hq,u}}{h^{h,hq}}$	Share of Hires of Graduates by High from U	28%	authors calculation	41%
$\frac{h^{h,hq,h}}{h^{h,hq}}$	Share of Hires of Graduates by High from High	55%	authors calculation	33%
$\frac{h^{h,hq,l}}{h^{h,hq}}$	Share of Hires of Graduates by High from Low	17%	authors calculation	26%
$\frac{h^{l,hq,u}}{h^{l,hq}}$	Share of Hires of Graduates by Low from U	27%	authors calculation	34%
$\frac{h^{l,hq,h}}{h^{l,hq}}$	Share of Hires of Graduates by Low from High	43%	authors calculation	46%
$\frac{h^{l,hq,l}}{h^{l,hq}}$	Share of Hires of Graduates by Low from Low	31%	authors calculation	20%
$\frac{h^{l,lq,u}}{h^{l,lq}}$	Share of Hires of Non-Graduates by Low from U	46%	authors calculation	48%
$\frac{h^{l,lq,l}}{h^{l,lq}}$	Share of Hires of Non-Graduates by Low from Low	54%	authors calculation	52%
Y	Aggregate Output	1	normalisation	0.98

Our model comprises 35 parameters. As discussed above, we calibrate $L^{hq} = 0.42$. We use the calibrations and estimates in Faccini et al. (2013) for the UK wherever a parameter of our model corresponds to a similar parameter in that study. Specifically, we follow Faccini et al. (2013) in setting the discount factor as $r = 0.101$, the opportunity cost of employment as $b = 0.58$, the coefficient of risk aversion in utility as $\eta = 0.73$ and the responses of monetary policy to inflation and the output gap as $\phi_\pi = 1.48$ and $\phi_y = 0.31$ respectively. We also assume the elasticities of matching with respect to unemployment for both high and low productivity firms are the same and set them to the elasticity in the aggregate matching function in Faccini et al. (2013), so $\alpha^h = \alpha^h = 0.3$. We assume $\nu^l = 11$. We also calibrate the bargaining power of workers in wage setting in the high productivity firms using the corresponding bargaining power in the aggregate wage bargain in Faccini et al. (2013), so $\zeta^h = 0.87$. For the parameters of the wage- and price-setting relationships, we follow a model developed by the Office of Budget Responsibility (Murray (2012)) and set $\kappa^h = \kappa^l = 0.1$ and $\varphi^{\pi,h} = \varphi^{\pi,l} = 0.85$. In the absence of previous calibrations for the UK, we set $\varphi^{w,h} = \varphi^{w,l} = 0.95$[21].

We calibrate the remaining 19 parameters to match our 20 calibration targets as closely as possible. We set the weight on high productivity retail goods in household utility as $\nu = 2$. We set the exogenous job destruction rates in high and low productivity firms as $\tau^h = 0.001$ and $\tau^l = 0.0125$, respectively. This implies that job matches in high productivity firms are much more likely to be terminated as the result of workers moving to other jobs than by workers moving to unemployment. We set the bargaining power of workers in wage setting in low productivity firms as $\zeta^l = 0.4$, so workers at low productivity firms have less than half the bargaining power of workers at high productivity firms. We set $m^h = 2.2$ and $m^l = 0.98$. The costs of

[21]We discuss this calibration further below

posting vacancies are calibrated as $\gamma^h = 0.4$ and $\gamma^l = 0.24$. The productivities of workers in high and low productivity firms are set as $A^h = 1.5$ and $A^l = 0.80$ respectively, so workers at high productivity firms are almost twice as productive as workers at low productivity firms. We assume $\nu^h = 20$. Finally, we calibrate the various search intensities as $\zeta^{h,hq,u} = 0.26$, $\zeta^{l,hq,u} = 0.14$, $\zeta^{l,lq,u} = 0.38$, $\zeta^{h,hq,h} = 0.009$, $\zeta^{h,hq,l} = 0.007$, $\zeta^{l,hq,h} = 0.006$, $\zeta^{l,hq,l} = 0.004$ and $\zeta^{l,lq,l} = 0.013$. These calibrations are summarised in Table 2). As the final column of Table 1) shows, our calibration enables us to match our calibration targets closely, although the match is less close for the complex pattern of transitions of workers between unemployment and between different jobs.

Our calibration implies that highly qualified workers find it more difficult to find employment than the less qualified, as $f^{h,hq} = 0.13$ and $f^{l,hq} = 0.14$ in steady-state, compared with $f^{l,lq} = 0.41$. High productivity firms fill their vacancies at a faster rate, as $q^h = 2.01$, compared to $q^l = 0.89$. Although it is more costly for high productivity firms to post vacancies, since $\gamma^h > \gamma^l$, the marginal cost of hiring workers is lower for high productivity firms, since $\frac{\lambda^h}{A^h} = 0.003$, compared to $\frac{\lambda^l}{A^l} = 0.012$, reflecting the faster rate at which high productivity firms fill their vacancies and the higher rate of job destruction at low productivity firms.

Table 2) Calibrated Parameters

Parameter	Interpretation	Source/Target	Value
L_{hq}	% Graduates in Labour Force	ONS data	0.420
r	Discount Rate	Faccini et al (2013)	0.0101
ϕ_π	Mon Pol Response to Inflation	Faccini et al (2013)	1.48
ϕ_y	Mon Pol Response to Output	Faccini et al (2013)	0.31
b	Opp Cost of Employment	Faccini et al (2013)	0.58
η	Risk Aversion	Faccini et al (2013)	0.73
α^h	Matching Elasticity for High Prod	Faccini et al (2013)	0.3
α^l	Matching Elasticity for Low Prod	Faccini et al (2013)	0.3
ν^l	Elasticity of Demand for Low Prod	Faccini et al (2013)	11
ζ^h	Bargaining Power for High Prod	Faccini et al (2013)	0.87
ν	% Weight on High Prod Goods in Utility	Authors' Calibration	2
τ^h	Exog Job Destuction in High Productivity Firms	Authors' Calibration	0.0011
τ^l	Exog Job Destuction in Low Productivity Firms	Authors' Calibration	0.0125
m^h	Matching Efficiency for High Productivity Firms	Authors' Calibration	2.15
m^l	Matching Efficiency for Low Productivity Firms	Authors' Calibration	0.8
ζ^l	Bargaining Power for Low Productivity Firms	Authors' Calibration	0.4
ν^h	Elasticity of Demand for High Prod	Authors' Calibration	20
γ^h	Vacancy Cost for High Productivity Firms	Authors' Calibration	0.4
γ^l	Vacancy Cost for Low Productivity Firms	Authors' Calibration	0.24
A^h	Productivity for High Productivity Firms	Authors' Calibration	1.5
A^l	Productivity for Low Productivity Firms	Authors' Calibration	0.80
$\zeta^{h,hq,u}$	Search by Unemp Grads for High Prod Jobs	Authors' Calibration	0.1
$\zeta^{l,hq,u}$	Search by Unemp Grads for Low Prod Jobs	Authors' Calibration	0.13
$\zeta^{l,lq,u}$	Search by Unemp Non-Grads for Low Prod Jobs	Authors' Calibration	0.355
$\zeta^{h,hq,h}$	Search by High Prod Grads for Other High Prod Jobs	Authors' Calibration	0.005
$\zeta^{h,hq,l}$	Search by Low Prod Grads for High Prod Jobs	Authors' Calibration	0.0025
$\zeta^{l,hq,h}$	Search by High Prod Grads for Low Prod Jobs	Authors' Calibration	0.007
$\zeta^{l,hq,l}$	Search by Low Prod Grads for Other Low Prod Jobs	Authors' Calibration	0.0034
$\zeta^{l,lq,l}$	Search by Low Prod Non-Grads for Other Low Prod Jobs	Authors' Calibration	0.015

4 Modelling the Pandemic

4.1 Modelling the Pandemic Using Shocks

We model the Covid-19 pandemic and the policy measures taken to mitigate this as a series of simultaneous shocks. To model the impact of these, we write the linearised representation of our model as

$$A_0 E_0 X_{k+1} = A_1 X_k + A_2 X_{k-1} + B\epsilon_k^{pan} \tag{53}$$

for $k = 1, 2, 3...$, where k is the number or quarters since the pandemic began, X_k is an $(n \times 1)$ vector containing the n endogenous variables of the model; ϵ_k^{pan} is a $(s \times 1)$ vector containing the s shocks that we use to represent the pandemic and policy measures; A_0, A_1 and A_2 are $(n \times n)$ matrices containing the structural parameters of the model, calibrated as decribed in the previous section; and B is an $(n \times s)$ matrix that captures the impact of the shocks on the endogenous variables. To simulate the pandemic, we first specify the shocks in ϵ^{pan}, as described below. We assume the pandemic began in 2020Q2, when $k = 1$; we assume the UK economy was in steady-state (relative to the major disruption that followed) in 2020Q1. We use a deterministic simulation of the model, showing the response of the endogenous variables to the shocks in ϵ^{pan}. We model the shocks as autoregressive processes, so

$$\varepsilon^z_{t+k} = \rho^z \varepsilon^z_{t+k-1} \tag{54}$$

where z indexes the shock; so the behaviour of the shock over time is characterised by the incidence in 2020Q2 and the persistence parameter. We begin by specifying a baseline simulation to show the likely impact over 2020-2023, based on information available in June 2020. We then consider alternative scenarios.

Considering how to model the shocks, we first note that the pandemic has reduced the number of workers. Although fatalities had a larger impact on less qualified workers, with the mortality for the top three occupational groups substantially below that for other groups, the distribution of self isolation has been more even (ONS (2020*a*)). The pandemic has also reduced aggregate demand, with sharp reductions in consumer spending and a a large reduction in the demand for consumer credit. There has been a marked reduction in consumer confidence. The impact on this has fallen more heavily on areas with a higher proportion of lower qualification workers, especially leisure, hospitality and entertainment. The pandemic has also led to a surge in job destruction, with almost two million additional claims for Universal Credit between mid-March and the first week of April (DWP (2020)); this was concentrated in lower qualification occupations. In addition, the pandemic has led to a rapid increase in the numbers working at home. Home working is heavily concentrated among those with higher qualifications (Costa Dias et al. (2020)), with 47% of graduates working at home in late April 2020, compared to around 15% of those with no qualifications (Gustafsson and McCurdy (2020)); related to this, over 50% of workers in managerial and professional occupations were working from home, compared to less than 10% in personal services, process and machine operatives and elementary occupations (*ibid*). Reflecting this, we model the pandemic as a combination of shocks: (i) $\varepsilon^{hq} < 0$ and $\varepsilon^{lq} < 0$, with $\varepsilon^{lq} < \varepsilon^{hq}$; (ii) $\varepsilon^{d} < 0$; (iii) $\varepsilon^{\Gamma^h} > 0$; (iv) $\varepsilon^{\tau^h} > 0$ and $\varepsilon^{\tau^l} > 0$, with $\varepsilon^{\tau^l} > \varepsilon^{\tau^h}$; and (v) $\varepsilon^{s^h} < 0$ and $\varepsilon^{s^l} < 0$, with $\varepsilon^{s^l} < \varepsilon^{s^h}$. These assumptions are sumarised in the first row of Table 3).

The public health response to the pandemic was a "lockdown", leading to the temporary closure of many workplaces and all shops, public spaces and schools, to restrict movements outside the home. We model this as a reduction in aggregate demand, with a larger impact on low productivity firms, reflecting the widespread closure of much of the entertainment and hospitality sectors[22]; also an increase in the rate of job destruction, with again a disproportionate effect on lower productivity firms; and a reduction in the productivity of workers, with a larger impact on the lower productivity sector, due to lower rates of home working. So we assume (i) $\varepsilon^{d} < 0$; (ii) $\varepsilon^{\Gamma^h} > 0$; (iii) $\varepsilon^{\tau^h} > 0$ and $\varepsilon^{\tau^l} > 0$, with $\varepsilon^{\tau^l} > \varepsilon^{\tau^h}$; and (iv) $\varepsilon^{s^h} < 0$ and $\varepsilon^{s^l} < 0$, with $\varepsilon^{s^l} < \varepsilon^{s^h}$. These assumptions are summarised in the second row of Table 3).

The adverse effects of these measures were to some extent offset by the Job Retention Scheme. Through this, the UK Treasury covered 80% of the cost of furloughed workers, up to a limit of £30,000 (slightly above 2019 UK annual median earnings of £29,400). By early May 2020, two thirds of UK firms had applied to the scheme and close to 7.5 million workers were on furlough. Take up was heavily skewed towards workers

[22]This response is consistent with the prescient simulation in Keogh-Brown et al. (2009)

with lower qualifications and working in lower productivity occupations (Leslie (2020)). The large number of workers on furlough preserved job matches and so reduced the rate of job destruction. By supporting the incomes of workers who would otherwise become unemployed, the Scheme also boosted aggregate demand. But the withdrawal of large numbers of employed workers from work led to a large decline in productivity. So we model a "Job Retention Scheme" as (i) $\varepsilon^d > 0$; (ii) $\varepsilon^{\tau^h} < 0$ and $\varepsilon^{\tau^l} < 0$, with $\varepsilon^{\tau^l} > \varepsilon^{\tau^h}$; (iii) $\varepsilon^{s^h} < 0$ and $\varepsilon^{s^h} < 0$, with $\varepsilon^{s^h} < \varepsilon^{s^h}$; and (iv) $\varepsilon^{w^l} < 0$ and $\varepsilon^{w^h} < 0$, with $\varepsilon^{w^l} < \varepsilon^{w^h}$. These assumptions are summarised in the third row of Table 3).

Other governmental responses to the pandemic also had a significant effect on the impact of the pandemic on the economy. In particular, there were large increases in state expenditure on health care, and large increases in Local Authority funding, used to finance a rapid transfer of patients from hospital into care homes, to make room for the rapid expansion in NHS capacity late March 2020, deferrals of VAT and other tax payments; the direct impact of these policy measures on cash borrowing in 2020-21 was forecast by the Office of Budget Responsibility in May 2020 to be £123 billion (OBR (2020)). We model the effect of this as an increase in aggregate demand and an increase in hiring in the low productivity sector reflecting the higher proportion of jobs requiring lower qualifications in the health and care sectors, that offset some of the loss of jobs in low productivity firms, so (i) $\varepsilon^d > 0$ and (ii) $\varepsilon^{\tau^l} < 0$. These effects are summarised in the fourth row of Table 3).

A strong monetary policy response from the Bank of England provided an additional mitigation to the adverse economic effects of the pandemic. From mid-March 2020, the Bank of England increased the size and scope of it's Quantitative Easing (QE) program, announcing additional purchases of £435bn, alongside other measures to support the financial system and to encourage lending (BoE (2020*b*)). We model the impact of this as a negative shock to the interest rate, so $\varepsilon^i < 0$. These effects are summarised in the fifth row of Table 3).

Table 3) Modelling the Impact of the Pandemic and Mitigating Policies via Shocks

shocks to	ε^{hq}	ε^{lq}	ε^d	ε^i	ε^{Γ^h}	ε^{τ_h}	ε^{τ_l}	$\varepsilon_t^{s^h}$	$\varepsilon_t^{s^l}$	$\varepsilon_t^{w^h}$	$\varepsilon_t^{w^l}$
Pandemic	↓	↓	↓		↑	↑	↑	↓	↓		
Public Health			↓		↑	↑	↑	↓	↓		
JRS			↑			↓	↓	↓	↓	↓	↓
Fiscal Policy			↑				↓				
Monetary Policy				↓							

4.2 Calibration of the Pandemic Shocks

We can classify the shocks in our simulation as being either aggregate supply or aggregate demand shocks, where the former comprise shocks to the workforce, to job destruction, to productivity and to wages. Aggregate demand shocks comprise shocks to monetary policy, to relative demand and to aggregate demand. Considering the aggregate supply shocks first, the main impact of the pandemic on the size of the workforce arose from workers being in self-isolation or sick with Covid-19. The impact of the pandemic on mortality of prime-age individuals in the UK was relatively limited, short in duration and mainly affected workers in health, care and transport occupations (ONS (2020*b*)). At the height of the pandemic in April 2020, 3% of workers were affected in this way, with a fairly even incidence across industries (Costa Dias et al. (2020)). So we calibrate ε_t^{hq} and ε_t^{lq} to generate a 3% reduction in L^{hq} and L^{lq}; we assume that this effect dissipates rapidly, reflecting the strong impact of the lock-down in reducing infection rates, by assumng $\rho^{hq} = \rho^{lq} = 0.05$. In this simulation, we assume there is no "second wave" of infections[23].

Information on the distribution of job losses by occupation is scarce at the time of writing. But the Resolution Foundation estimated job losses by industry in April 2020 (Tomlinson (2020)), finding that these are concentrated in areas such as Hospitality, Retail and Construction, with high proportions of employment of lower qualification workers. This is partly offset by an increase in employment in the Health and Care sectors. By contrast, job losses in sectors with higher concentrations of more highly qualified workers, such as Finance and Insurance and Public Administration, are 5-6 times lower. Based on this evidence, we calibrate $\varepsilon_t^{\tau^l}$ so that the rate of exogenous job destruction in low productivity firms increases to 6.7% at the onset of the crisis. This generates an increase of 1.2 million lower qualification workers and 200,000 higher qualification workers becoming unemployed. We calibrate $\varepsilon_t^{\tau^h}$ so that the rate of exogenous job destruction in high productivity firms increases to 2.9% in 2020Q2; this generates an additional flow of 200,000 newly unemployed higher qualification workers. We assume $\rho^{\tau^l} = \rho^{\tau^h} = 0.5$, so the wave of job losses dies away quite rapidly. Together, these shocks lead to an additional 1.6 million job losses at the onset of the pandemic[24].

To construct the supply shocks for the representative high productivity firms, we define the effective workforce as $n_t^{h,e} = (1 - \pi^{f,h}\omega_t^{f,h} - \pi^{wfh,h}\omega_t^{wfh,h})n_t^h$, where $\omega_t^{f,h}$ and $\omega_t^{wfh,h}$ are the proportions of workers at high priductivity firms who are furloughed and working from home respectively, and $\pi^{wfh,h}$ and $\pi^{wfh,h}$ are the relative productivities of these workers. Since output is $Y_t^h = A_t^h n_t^{h,e}$, output per employed worker is $\frac{A_t^h n_t^{h,e}}{n_t^h} = (1 - \pi^{f,h}\omega_t^{f,h} - \pi^{wfh,h}\omega_t^{wfh,h})A_t^h$. We assume that furloughed workers do not contribute to output, so $\pi^{wfh} = 0$ and that working from home reduces productivity by 10%, so $\pi^{wfh} = 0.9$. We use occupational-level data on the numbers of workers furloughed and working at home in April 2020, constructed by Gustafsson and McCurdy (2020). Using ONS data on employment by occupation, we use these data to construct measures of the share of workers employed in high productivity firms who were furloughed or working from home in April 2020. We adjusted the numbers furloughed to reflect increased take-up of the Job Retention Scheme (JRS) until June 2020, to get estimates for 2020Q2[25]; we find that over 2020Q2, an average of 19.4% of workers at high productivity firms were furloughed, so $\omega_t^{f,h} = 0.194$, and 51.5% of workers at high productivity firms worked from home, so $\omega_t^{wfh,h} = 0.515$. Productivity per employed worker at high productivity firms in 2020Q2 is then (100-19.4-0.1*51.5)=75.5% of the pre-pandemic level. We calibrate the high productivity supply shock so that $e^{\varepsilon_t^{s^h}} = (1 - \pi^{f,h}\omega_t^{f,h} - \pi^{wfh,h}\omega_t^{wfh,h})$ matches this figure. We also find that over 2020Q2, an average of 21% of workers at low productivity firms were furloughed and 17% were working from home. Productivity per employed worker at the representative low productivity firm in 2020Q2 is then (100-21.4-0.1*17)=77.1% of the pre-pandemic level. We calibrate the low productivity supply shock to match this. We assume $\rho^{s^h} = \rho^{s^l} = 0.25$.

To calculate the impact of the Job Retention Scheme on wages, we first consider the representative low productivity firm. We express the wages paid by this firm as $\{\omega_t^{f,l}(1 - \phi_t^{f,l}) + (1 - \omega_t^{f,l})\}w_t^l$, where $\phi_t^{f,l}$ is the share of the wage paid by the state under the JRS. The Job Retention Scheme paid 80% of the wages of furloughed workers, up to a limit of £30,000, close to the median wage; so we assume $\phi_t^{f,l} = 0.8$. This

[23] Coibion et al. (2020) have suggested that the pandemic led to the withdrawal of workers from the workforce in the US. The surge in job search activity by new claimants for Universal Credit suggests this effect is small in the UK, Brewer and Handscomb (2020).

[24] This is somewhat above the number of new claims for Universal Credit in this period, but not all workers are eligible for Universal Credit.

[25] The number of workers furloughed using the JRS increased from 3.8 million in April 2020 to 8.9 million in June 2020. For 2020Q2 as a whole, we used the average of these figures. We assumed that the numbers working from home did not change between April-June 2020.

implies that the wage cost to the representative low productivity firm is 21*0.2+79 or 83.2% of the wage. We calibrate the wage shock for low productivity firms to match this. We express the wages paid by the representative high productivity firm as $\{\omega_t^{f,h}(1-\phi_t^{f,h})+(1-\omega_t^{f,h})\}w_t^h$, where $\phi_t^{f,h}$ is the share of the wage paid by the state under the JRS. The calculation here is less straightforward since the wage in these firms will be above the median. Here we assume that the JRS pays 40% of the wage, so $\phi_t^{f,l}=0.4$.. This implies that the wage cost to the representative high productivity firm is 19.4*0.6+80.6 or 92.2% of the wage. We calibrate the wage shock for high productivity firms to match this. Reflecting the short duration of the JRS, we assume $\rho^{w^h}=\rho^{w^l}=0.15$.

To model the impacts of the pandemic on aggregate demand, we calibrate ε^i so that the (shadow) interest rate decreases by 250 basis points in 2020Q2; we assume $\rho^i=0.75$, so this effect is relatively persistent. Initial estimates of GDP for April 2020 suggest much sharper falls in output in hospitality and retail industries, with a lower fall in output that makes greater use of more highly qualified workers. To model this, we calibrate ε^{Γ^h} so that the fall in demand for the output of high productivity firms is 25% smaller than the fall in demand for the output of low productivity firms. Finally, we calibrate the aggregate demand shock ε^d to match the projected reduction in UK GDP in June 2020. Here, there is little consensus. The May 2020 Bank of England *Monetary Policy Report* projects a reduction of 14% in the main illustrative baseline scenario (BoE (2020*a*)). The OECD expects a falls of 11.4% (OECD (2020)), while the IMF projects a fall of 10.2% (IMF (2020)). We calibrate ε^d to generate a fall in UK GDP over 2020 of 11%.

Table 4) The Baseline And Other Scenarios

Impact	(i) Baseline	(ii) Scenario 1	(iii) Scenario 2	(iv) Scenario 3	Scenario 4
L^{hq}	$\downarrow$ 3%	$\downarrow$ 3%	$\downarrow$ 9%	$\downarrow$ 3%	$\downarrow$ 3%
L^{lq}	$\downarrow$ 3%	$\downarrow$ 3%	$\downarrow$ 9%	$\downarrow$ 3%	$\downarrow$ 3%
τ^h	$\uparrow$ 168%	$\uparrow$ 168%	$\uparrow$ 230%	$\uparrow$ 168%	$\uparrow$ 168%
τ^l	$\uparrow$ 335%	$\uparrow$ 168%	$\uparrow$ 425%	$\uparrow$ 335%	$\uparrow$ 335%
A^h	$\downarrow$ 24.5%	$\downarrow$ 24.5%	$\downarrow$ 5.2%	$\downarrow$ 24.5%	$\downarrow$ 24.5%
A^l	$\downarrow$ 22.9%	$\downarrow$ 24.5%	$\downarrow$ 1.7%	$\downarrow$ 22.9%	$\downarrow$ 22.9%
w^h	$\downarrow$ 7.8%	$\downarrow$ 7.8%		$\downarrow$ 7.8%	$\downarrow$ 7.8%
w^l	$\downarrow$ 17.0%	$\downarrow$ 7.8%		$\downarrow$ 17.0%	$\downarrow$ 17.0%
i	$\downarrow 250bp$	$\downarrow 250bp$	$\downarrow 250bp$	$\downarrow 250bp$	$\downarrow 250bp$
$\frac{Y^h}{Y^l}$	$\uparrow$ 25.0%		$\uparrow$ 25.0%	$\uparrow$ 25.0%	$\uparrow$ 25.0%
Y^d	$\downarrow$ 11.0%	$\downarrow$ 11.0%	$\downarrow$ 13.2%	$\downarrow$ 11.0%	$\downarrow$ 11.0%

The persistence of shocks is altered in Scenario 3. In Scenario 4, the shocks of 2020Q2 recur in 2020Q4

4.3 Baseline Scenario

The calibration of shocks in our baseline simulation, as outlined above, are contained in column (i) of Table 4). The results of our baseline simulation is shown in Figure 1). We begin by considering the aggregate impacts. Our model projects a relatively quick recovery in output, which returns to pre-pandemic levels by mid-2021. The recovery in other variables is slower. Unemployment increases to 2.97 million workers, an unemployment rate of 8.7%, by 2020Q3; this is close to the forecast of 9% in the Bank of England's *Monetary Policy Report* of May 2020 (BoE (2020*a*)).. The recovery in the unemployment rate is slow; it does not return to pre-pandemic levels until 2022. Employment falls by 6.6% by the end of 2020 and remains below pre-pandemic values until 2021. The real wage falls by 0.9%; this recovers quickly and exceeds the pre-pandemic level by early 2022[26]. Our scenario projects deflation of -2% in 2020, followed by several years of above-target inflation. rising to 4% during the recovery from the pandemic. However, this should be treated with some caution[27].

Turning to the main focus of our paper, our simulations reveal that the pandemic has very different impacts on different types of worker. Job losses affect non-graduates much more severely than graduates. Nearly 1.2 million non-graduates lose their jobs by the end of 2020, compared to 0.4 million job losses among graduates. The unemployment rate of non-graduates rises to 10.1% compared to 6.8% for graduates; by the end of 2020Q3, 2.0 million non-graduates are unemployed, compared to just below 1 million graduates. The real wages of non-graduates fall by 2.0%, whereas wages of graduates employed in high productivity firms fall by 1.9%. The employment of different types of worker differs markedly across the pandemic. Employment of non-graduates in low productivity firms falls by 8.1% by the end of 2020 and reaches pre-pandemic values by mid-2021. The structure of graduate employment changes markedly and the adjustment back to previous values lasts beyond 2024. Employment of graduates in high productivity firms declines slightly but then increases, driven by the increased demand for the output of high productivity firms. By contrast, employment of graduates in non-graduate roles falls 9.6%. Employment of graduates in these different types of firm only returns slowly to pre-pandemic levels, with the adjustment lasting beyond 2023. Interestingly, our model predicts a rise in productivity as the proportion of graduates employed in lower productivity firms declines. The different experience of different workers gives rise to composition effects on the aggregate wage, which falls by less than the wages paid by high productivity and low productivity firms, due to the rise in the share of employment at higher wage high productivity firms. Without this composition effect, the real wage would fall by 1.9%. Similarly, the shift in consumption towards the output of high productivity firms moderates the fall in inflation; the inflation rate would reach -2.9% without this composition effect.

4.4 Scenario 1: Shocks or Structure?

The labour market experience of non-graduates during the pandemic is clearly worse than that of graduates. This reflects both the structure of the economy, since graduates employed by high productivity firms have higher wages and greater job security than non-graduates, and the nature of the shocks caused by the pandemic, which concentrated job losses in low productivity firms and led to larger falls in demand for these firms. To study the relative impact of these, we construct a counterfactual scenario in which the adverse shocks affecting high and low productivity firms are equal. To do this, we amend the shocks as in column (ii) of Table 4), so the impact of the pandemic on job destruction, productivity and wages is the same for high and low productivity firms, and there is no shift in demand towards the output of high productivity firms.

The results, shown in Figure 2), reveal how the shocks associated with the pandemic have increased the disadvantage of non-graduates in the UK labour market. Unemployment of non-graduates continues to exceed unemployment of graduates, but the gap between them narrows. Unemployment of non-graduates fall from 2.18 million to 1.72 million. The percentage fall in employment of non-graduates is now similar to that of graduates employed in high productivity firms; in contrast to the baseline scenario. The proportional fall in wages in high and low productivity firms is now also more similar, with non-graduate wages falling

[26]If we use $\varphi^{w,h} = \varphi^{w,l} = 0.85$, the real wage falls by 5.4%.

[27]For two reasons; first, inflation is especially difficult to forecast during the pandemic as adverse demand and supply shocks move it in different directions, in contrast to the effect of these shocks on output, employment and unemployment; and second, our simple model does not allow for variations in the mark-up. There is evidence that increased mark-ups increased inflation from 2020Q2, as major retailers ceased competitive price reductions (Jaravel and O'Connell (2020)).

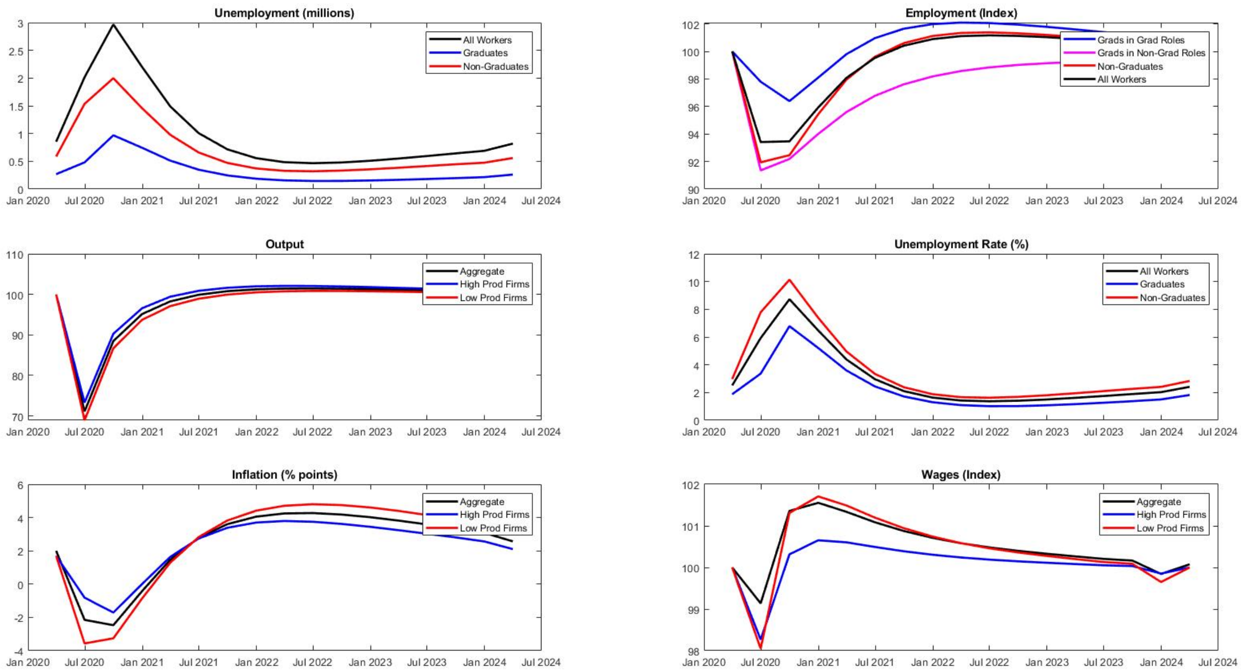

Figure 1: The Baseline Model

Notes: The top left panel of this figure plots the numbers of unemployed workers (solid black line), the number of unemployed non-graduates (solid red line) and the number of unemployed graduates in the baseline scenario (solid blue line). The top right panel plots indices for total employment (solid black line), graduates in high productivity firms (solid blue line), graduates in low productivity firms (solid pink line) and non-graduates in low productivity firms in the baseline (solid red line). The middle left panel plots indices for total output (solid black line), output of high productivity firms (solid blue line and output of low productivity firms (solid red line). The middle left panel plots the unemployment rate of all workers (solid black line), graduates (solid blue line) and non-graduates (solid red line). The bottom left panel plots the aggregate inflation rate (solid black line) and the inflation rates of high productivity firms (solid blue line) and low productivity firms (solid red line). The bottom right panel plots indices for the aggregate wage (solid black line) and real wages in high productivity firms (solid blue line) and low productivity firms (solid red line).

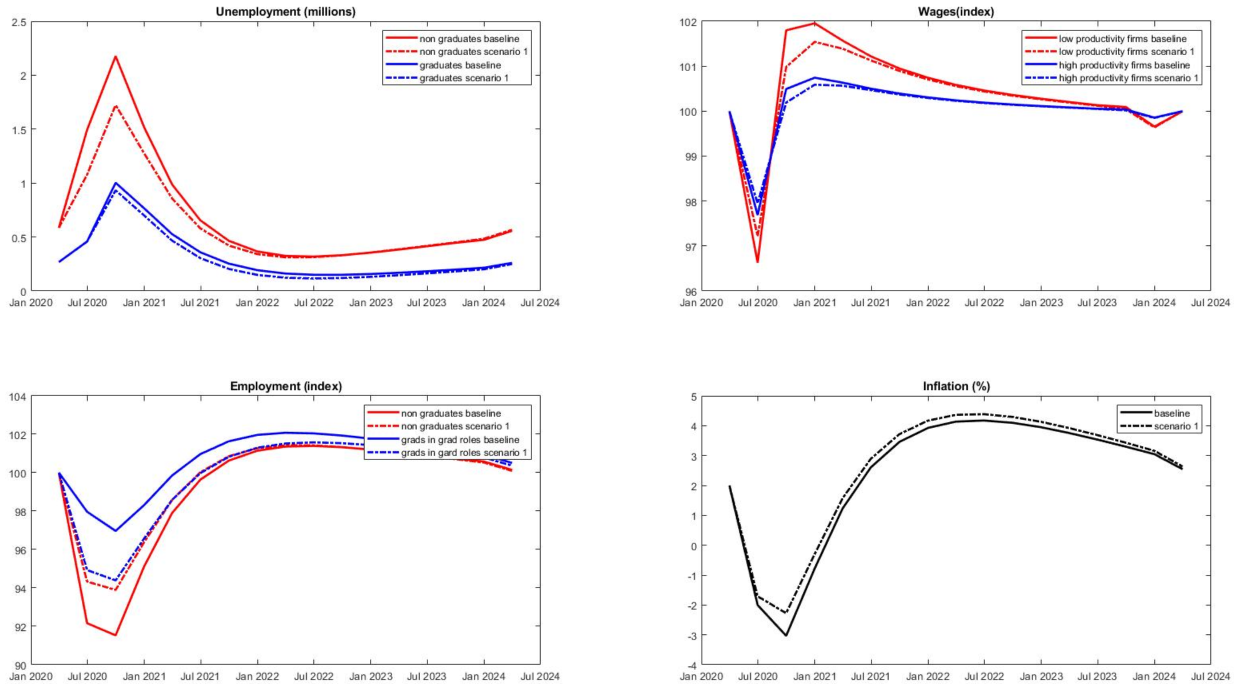

Figure 2: Scenario 1: The Same Shocks

Notes: The top left panel of this figure plots the number of unemployed non-graduates in the baseline scenario (solid red line) and the scenario (dotted red line) and the number of unemployed graduates in the baseline scenario (solid blue line) and the scenario (dotted blue line). The top right panel plots wages of high productivity firms in the baseline (solid blue line) and the scenario (dotted blue line) and wages of low productivity firms in the baseline (solid red line) and the scenario (dotted red line). The bottom left panel plots employment of graduates in high productivity firms in the baseline (solid blue line) and the scenario (dotted blue line) and employment of non-graduates in low productivity firms in the baseline (solid red line) and the scenario (dotted red line). The bottom right panel plots the inflation rate in the baseline (solid black line) and the scenario (dotted black line).

by 2.8%, compared to 3.4% in the baseline. As a consequence, the rate of deflation is reduced.

4.5 Scenario 2: No "Job Retention Scheme"

In this section, we present a simple, preliminary illustration of the impact of the simple "Job Retention Scheme" (JRS) used in our baseline simulation. To do this, we constructed a scenario in which Government support for wages was removed, so $\varepsilon^{w^h} = \varepsilon^{w^l} = 0$. We assume that 25% of workers who were furloughed would have been made redundant in the absence of the scheme. This implies additional job losses of 25%*21.4%*$(n^{l,hq} + n^{l,lq})$, or 1.28 million workers in low productivity firms and 25%*19.4%*$n^{h,hq}$, or 0.41 million workers in high productivity firms. Productivity per employed worker at high productivity firms in 2020Q2 is this scenario is then (100-0.1*51.5)=94.8% of the pre-pandemic level, while productivity at low productivity firms (100-0.1*17)=98.3% of the pre-pandemic level. Since social distancing is more difficult for workers at the workplace than at home, we assume the number of workers infected or in self isolation triples. The increased job losses also impact on aggregate demand, so we assume that the aggregate demand shock is 20% worse than in the baseline scenario.

The results, in Figure 3), show that the "Job Retention Scheme" had a major impact. Unemployment

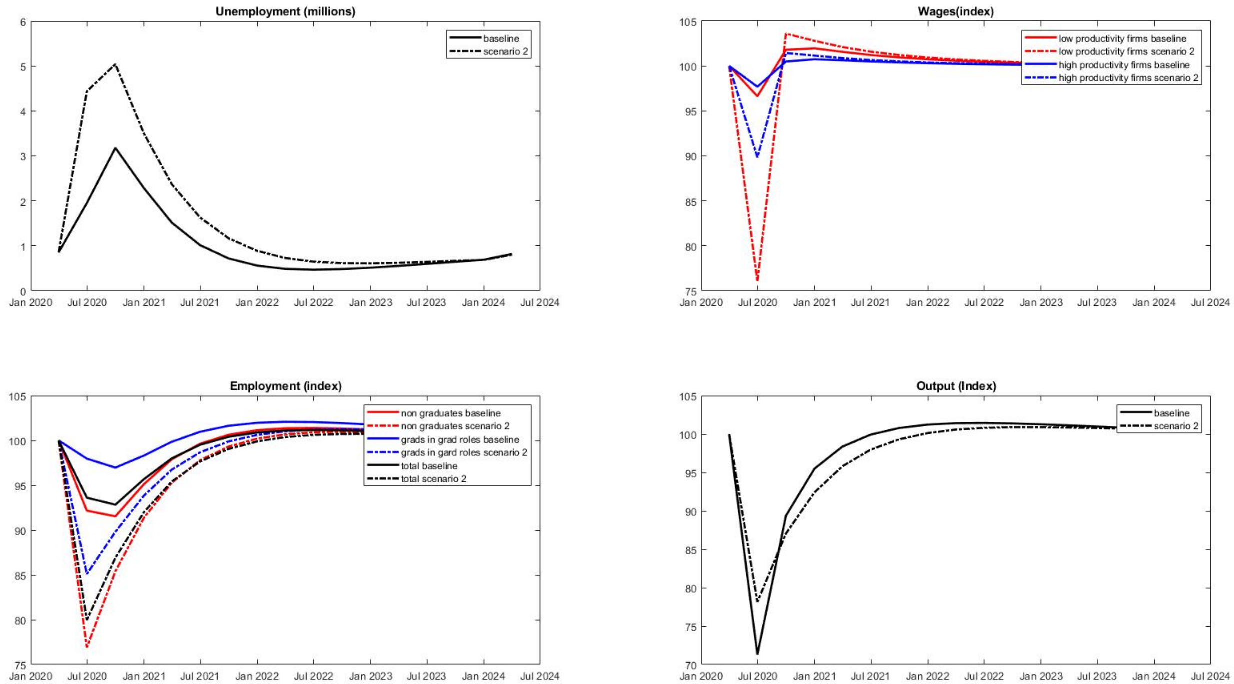

Figure 3: Scenario 2: The "Job Retention Scheme"

Notes: The top left panel of this figure plots the number of unemployed workers in the baseline scenario (solid black line) and the scenario (dotted black line). The top right panel plots wages of high productivity firms in the baseline (solid blue line) and the scenario (dotted blue line) and wages of low productivity firms in the baseline (solid red line) and the scenario (dotted red line). The bottom left panel plots total employment in the baseline (solid black line) and the scenario (dotted black line); employment of graduates in high productivity firms in the baseline (solid blue line) and the scenario (dotted blue line) and employment of low productivity workers in the baseline (solid red line) and the scenario (dotted red line). The bottom right panel plots output in the baseline (solid black line) and the scenario (dotted black line).

rises to 5.04 million, with 3.29 milion unemployed non-graduates and 1.77 million unemployed graduates. The unemployment rate reaches 14.82%, with rates of 16.69% and 12.23% for non-graduates and graduates respectively. Employment falls by 20%, with falls of 23% for non graduates and by 15% for graduates. The real wages of non-graduates fall by 24.0%, while the real wages of graduates fall by 11.1%. The average wage falls by 15.5%. Despite this, the fall in output is less severe, since the productivity of employed workers is higher.

4.6 Scenario 3: A Jobs Package

In this section, we model the impact of a "jobs package", designed to reduce job destruction and accelerate job creation, through a slower removal of the Job Retention Scheme and the introduction of subsidies to vacancy posting. Slower removal of the JRS increases the persistence of negative supply shocks and the wage subsidies and lowers the persistence of job destruction shocks. To model this, we reduce ρ^{τ^l} and ρ^{τ^h} from 0.5 to 0.25, we increase ρ^{s^h} and ρ^{s^l} from 0.25 to 0.5 and increase ρ^{w^h} and ρ^{w^l} from 0.15 to 0.5. We also assume shocks to the cost of vacancy posting; at the onset of the pandemic, vacancy costs are halved, so that the cost of posting a vacancy for a high productivity firm is reduced from 0.4 to 0.2 and the cost of posting a vacancy for a low productivity firm is reduced from 0.24 to 0.12. The persistence of these shocks

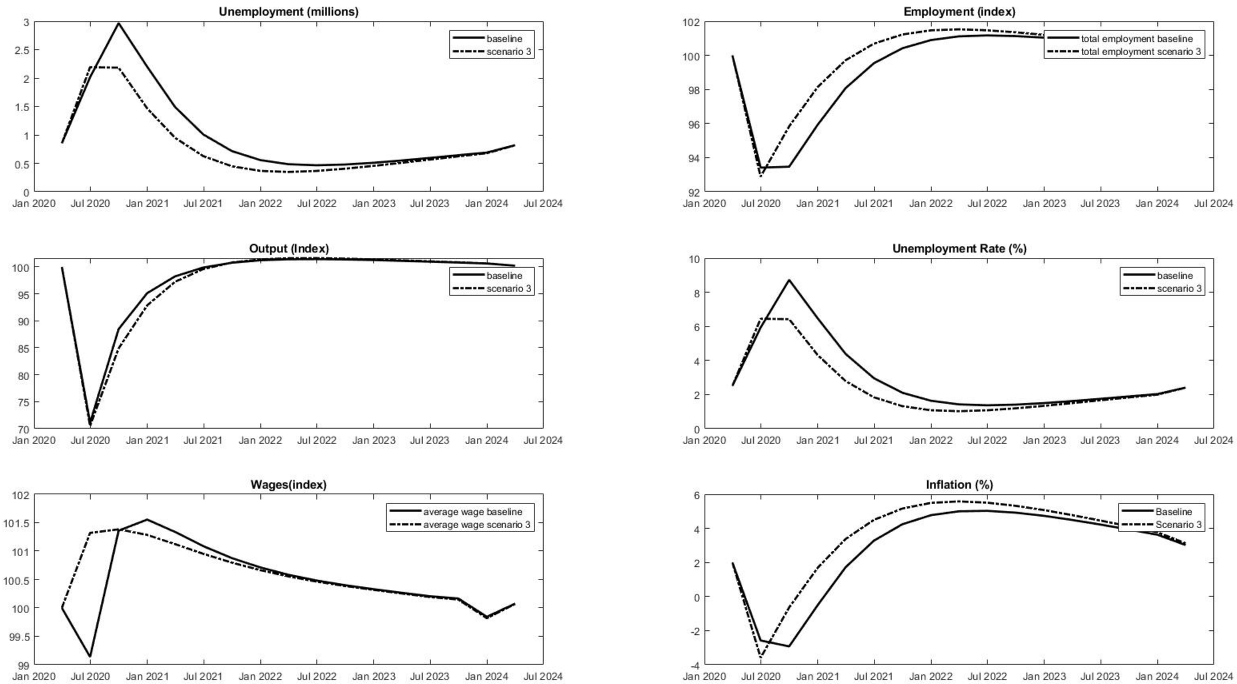

Figure 4: Scenario 3: A "Jobs Package"

Notes: The top left panel of this figure plots the number of unemployed workers in the baseline scenario (solid black line) and the scenario (dotted black line). The top right panel plots the total employment in the baseline scenario (solid black line) and the scenario (dotted black line). The middle left panel plots output in the baseline scenario (solid black line) and the scenario (dotted black line). The middle right plots the aggregate unemployment rate in the baseline scenario (solid black line) and the scenario (dotted black line). The bottom left panel plots the aggregate wage in the baseline scenario (solid black line) and the scenario (dotted black line). The bottom right plots the inflation rate in the baseline scenario (solid black line) and the scenario (dotted black line).

is set as 0.5.

The results, in Figure 4), show that the "Jobs Package" has a positive effect. Unemployment rises to 2.2 million, compared to 3 million in the baseline scenario. Unemployment of graduates and non-graduates are both around 400,000 lower. The rise in the unemployment rate peaks at 6.4%. The initial fall in employment is similar to the baseline, but the recovery in employment is accelerated by around 3 months. The fall in real wages is avoided, as wages increase by 1.4%. The initial deflation is similar to the baseline, but deflation disappears around 3 months earlier. Thereafter, inflation is around 1% higher than in the baseline.

4.7 Scenario 4: A Second Wave

In this section, we assume there is an anticipated "second wave" of the pandemic, in 2020Q4. We assume that the shocks in 2020Q2 and 2020Q3 are the same as the baseline. The second wave reinstates the values of the shocks of 2020Q2 in 2020Q4. So $\varepsilon^z_{t+k} = \rho^z \varepsilon^z_{t+k-1}$ for $k = 0, 1$; $\varepsilon^z_{t+2} = \varepsilon^z_t$; and $\varepsilon^z_{t+k} = \rho^z \varepsilon^z_{t+k-1}$ for $k = 3, 4, ...$ The results, in Figure 5), highlight the potential damage from a second wave. The loss of output is larger than in the baseline, and the recovery reverses when the second wave hits. The recovery of output is much slower; ouput now only returns to pre-pandemc levels in mid-2022. Unemployment rises to 4.5 million, an unemployment rate of 13.25%. Employment falls by 12%, but the recovery in employment is not delayed.

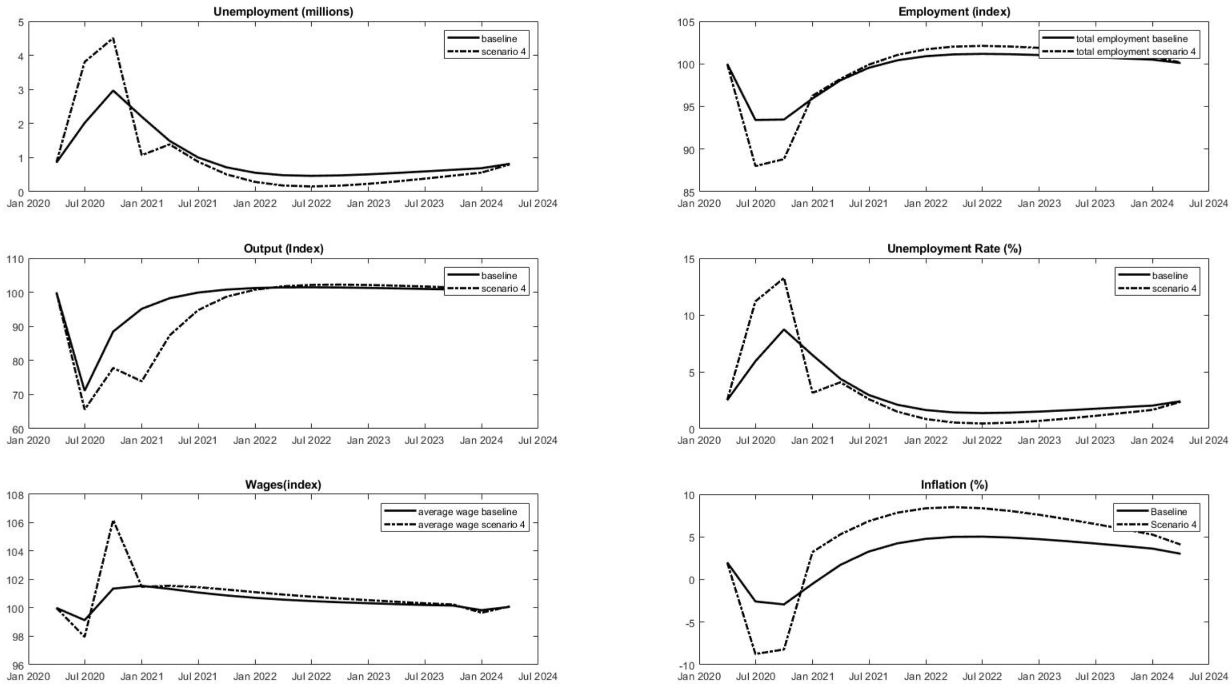

Figure 5: Scenario 4: A "Second Wave"

Notes: The top left panel of this figure plots the number of unemployed workers in the baseline scenario (solid black line) and the scenario (dotted black line). The top right panel plots the total employment in the baseline scenario (solid black line) and the scenario (dotted black line). The middle left plots output in the baseline scenario (solid black line) and the scenario (dotted black line). The middle right panel of this figure plots the aggregate unemployment rate in the baseline scenario (solid black line) and the scenario (dotted black line). The bottom left panel plots the aggregate wage in the baseline scenario (solid black line) and the scenario (dotted black line). The bottom right plots the inflation rate in the baseline scenario (solid black line) and the scenario (dotted black line).

Real wages fall by 2%, but then recover rapidly as firms seek to rebuild their workforces. The second wave leads to much deeper deflation, floowed by several years of high inflation.

5 Conclusions

This paper has analysed the differing impacts of the Covid-19 pandemic on graduates and non-graduates. We have highlighted how the crisis will exacerbate structural differences in the UK labour market as job loss and unemployment will disproportionately affect non-graduates and lead to sharper falls in the real wages of these workers. We have argued that the Job Retention Scheme will have a major impact on the UK labour market, by reducing job losses and lessening the fall in wages. We have further argued that a "Jobs Package", that extends the JRS and subsidises job creation would further cushion the impact of the pandemic on the labour market, and would shorten the recovery period.

Our results show that a standard DSGE model with labour market frictions can give useful insights into the impact of the pandemic and can be useful for policy evaluation. But our results are dependent on a specific model that makes a series of assumptions. Given the unprecedented scale and nature of the crisis, it is important that the impact of the pandemic on the labour market is also analysed using alternative approaches, which address the impact of uncertainty and which incorporate self-employment and individuals who are out-of-the workforce. We hope to extend our analysis to address these issues.

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7 Appendix: Wage Determination

7.1 Wage Determination in the Low Productivity Wholesale Firm

We assume that wage bargaining takes place between the firm and low qualification workers[28]. The bargained wage is determined by the sharing rule

$$(1-\zeta^l)S_t^{l,lq} = \zeta^l F_t^l \tag{55}$$

where $S_t^{l,lq}$ is the surplus to the household from an additional low qualification worker being employed in a low productivity firm, F_t^l is the surplus to the firm and ζ^l is the bargaining power of low qualification workers in the low productivity sector.

[28] Although high qualification and low qualification workers have the same productivity and must be paid the same wage, a match with a low qualification worker has a different value to a low productivity firm than a match with a high qualification worker, because the respective matches break down with different probabilities.

The firm's surplus is $F_t^{l,lq} = \frac{\partial J_t^l}{\partial N_t^{l,lq}}$. Because of the assumption of constant returns, we can combine this with the optimality condition for low productivity firms to obtain

$$F_t^l = \frac{P_t^{l,W}}{P_t^l} A_t^l - w_t^{b,l} + E_t \rho_t^{l,lq} \frac{\gamma^l}{q_{t+1}^l} \tag{56}$$

and

$$E_t F_{t+1}^l = \frac{\gamma^l}{E_t q_{t+1}^l} \tag{57}$$

The surplus the household derives from successful conclusion of the wage bargain, which is employment of an additional unskilled household member at the low productivity firm less the outside option of that worker, which is being unemployed. So

$$S_t^{l,lq} = \frac{1}{C_t^{-\gamma}} \left(\frac{\partial H_t}{\partial n_t^{l,lq}} - \frac{\partial H_t}{\partial u_t^{lq}} \right) \tag{58}$$

Since $\frac{\partial H_t}{\partial n_t^{l,un}} = C_t^{-\gamma} w_t^{b,l} + \rho_t^{l,lq} \frac{\partial H_t+1}{\partial n_{t+1}^{l,lq}}$ and $\frac{\partial H_t}{\partial u_t^{lq}} = C_t^{-\gamma} b_t + f_{t+1}^{l,lq,u} \frac{\partial H_{t+1}}{\partial n_{kt+1}^{l,lq}}$, where k denotes an alternative low productivity firm, the successful conclusion of the wage bargain implies that the household gains an additional unskilled worker employed at a low productivity firm; this persists into the next period with probability $\rho_t^{l,lq}$. But the household loses an unemployed low qualification member, who would have moved into alternative employment in the low productivity sector at the beginning of the next period with probability $f_{t+1}^{l,lq,u}$. This implies

$$S_t^{l,lq} = w_t^{b,l} - b + (\rho_t^{l,un} - f_{t+1}^{l,lq,u}) E_t \beta_{t,t+1} S_{t+1}^{l,lq} \tag{59}$$

The sharing rule in (55) implies that the household surplus can be written as

$$\zeta^l F_t^l = (1 - \zeta^l)(w_t^{b,l} - b) + \zeta^l E_t \beta_{t,t+1} (\rho_t^{l,lq} - f_{t+1}^{l,lq,u}) F_{t+1}^l \tag{60}$$

Using (56) and (57) respectively, we obtain

$$\zeta^l \left\{ \frac{P_t^{l,W}}{P_t^l} A_t^l - w_t^{b,l} + E_t \rho_t^{l,lq} \beta_{t,t+1} \frac{\gamma^l}{q_{t+1}^l} \right\} = (1 - \zeta^l)(w_t^{b,l} - b) + \zeta^l E_t \beta_{t,t+1} (\rho_t^{l,lq} - f_{t+1}^{l,lq,u}) \frac{\gamma^l}{q_{t+1}^l} \tag{61}$$

or

$$w_t^{b,l} = \zeta^l \left\{ A_t^l + \zeta^l E_t E_t \beta_{t,t+1} \theta_{t+1} \right\} + (1 - \zeta^l) b \tag{62}$$

Using $f_t^{l,lq,u} = \zeta^{l,lq,u} f_t^l$, we obtain

$$w_t^{b,l} = \zeta^l \left\{ \frac{P_t^{l,W}}{P_t^l} A_t^l + \gamma^l \zeta^{l,lq,u} E_t \beta_{t,t+1} \theta_{t+1}^l \right\} + (1 - \zeta^l) b \tag{63}$$

7.2 Wage Determination in the High Productivity Wholesale Firm

We assume that wages for high productivity jobs are determined through Nash bargaining between high productivity wholesale firms and individual high qualification workers. The bargained wage is chosen to maximise

$$S_t = (S_t^{h,hq})^{\zeta^h} (F_t^h)^{1-\zeta^h} \tag{64}$$

where $S_t^{h,hq}$ is the surplus to the household from an additional worker being employed in a high productivity firm, F_t^h is the surplus to the firm and ζ^h is the bargaining power of high qualfication workers in high productivity jobs. This gives the sharing rule

$$(1-\zeta^h)S_t^{h,hq} = \zeta^h F_t^h \tag{65}$$

The surplus for a representative high productivity firm is

$$F_t^h = \frac{\partial J_t^h}{\partial n_t^{h,hq}} \tag{66}$$

Combining this with the optimality conditions gives

$$F_t^h = \frac{P_t^{h,W}}{P_t^h} A_t^h - w_t^{b,h} + E_t \rho_t^h \frac{\gamma^h}{q_{t+1}^h} \tag{67}$$

and

$$E_t F_{t+1}^h = \frac{\gamma^h}{E_t q_{t+1}^h} \tag{68}$$

The surplus the household derives from successful conclusion of the wage bargain, which is employment of a high qualification household member at the high productivity firm less the outside option of that worker, which is being unemployed. So

$$S_t^{h,hq} = \frac{1}{C_t^{-\gamma}} \left(\frac{\partial H_t}{\partial n_t^{h,hq}} - \frac{\partial H_t}{\partial u_t^{hq}} \right) \tag{69}$$

The successful conclusion of the wage bargain implies that the household gains an additional high qualification worker employed at a high productivity firm; this match persists into the next period with probability ρ_t^h. But the household loses an unemployed high qualification member, who would have moved into alternative employment in the high productivity sector with probability $f_{t+1}^{h,hq,u}$ or alternative employment in the low productivity sector with probability $f_{t+1}^{l,hq,u}$. This implies $\frac{\partial H_t}{\partial n_t^{h,hq}} = C_t^{-\gamma} w_t^{b,h} + \rho_t^h \frac{\partial H_t+1}{\partial n_{t+1}^{h,hq}}$ and $\frac{\partial H_t}{\partial u_t^{hq}} = C_t^{-\gamma} b_t + f_{t+1}^{h,hq,u} \frac{\partial H_{t+1}}{\partial n_{kt+1}^{h,hq}} + f_{t+1}^{l,hq,u} \frac{\partial H_{t+1}}{\partial n_{k't+1}^{l,hq}}$, where k here denotes an alternative high productivity firm and k' denotes a low productivity firm. As all high productivity firms are identical and all low productivity firms are identical, this implies

$$S_t^{h,hq} = w_t^{b,h} - b + E_t \beta_{t,t+1} (\rho_t^h - f_{t+1}^{h,hq,u}) S_{t+1}^{h,hq} - E_t \beta_{t,t+1} f_{t+1}^{l,hq,u} S_{t+1}^{l,sk} \tag{70}$$

The sharing rules in (55) and (65) imply that the household surplus can be written as

$$\zeta^h F_t^h = (1-\zeta^h) w_t^{b,h} - b + \zeta^h E_t \beta_{t,t+1} (\rho_t^h - f_{t+1}^{h,hq,u}) F_{t+1}^h - \zeta^h E_t \beta_{t,t+1} f_{t+1}^{l,hq,u} F_{t+1}^l \tag{71}$$

Using (57), (67) and (68), we obtain

$$\zeta^h \left\{ \frac{P_t^{h,W}}{P_t^h} A_t^h - w_t^{b,h} + E_t \beta_{t,t+1} \rho_t^h \frac{\gamma^h}{q_{t+1}^h} \right\} = (1-\zeta^h)(w_t^{b,h} - b) + \zeta^h E_t \beta_{t,t+1} (\rho_t^h - f_{t+1}^{h,hq,u}) \frac{\gamma^h}{q_{t+1}^h} - \zeta^h E_t \beta_{t,t+1} f_{t+1}^{l,sk,u} \frac{\gamma^l}{q_{t+1}^l} \tag{72}$$

Simplifying this

$$w_t^{b,h} = \zeta^h \left\{ \frac{P_t^{h,W}}{P_t^h} A_t^h + \gamma^h E_t \theta_{t+1}^h + \gamma^l E_t \theta_{t+1}^l \right\} + (1-\zeta^h) b \tag{73}$$

Noting that $f_t^{h,hq,u} = \zeta^{h,hq,u} f_t^h$ and $f_t^{l,hq,u} = \zeta^{l,hq,u} f_t^l$ gives

$$w_t^{b,h} = \zeta^h \left\{ \frac{P_t^{h,W}}{P_t^h} A_t^h + \gamma^h \zeta^{h,hq,u} E_t \theta_{t+1}^h + \gamma^l \zeta^{l,hq,u} E_t \theta_{t+1}^l \right\} + (1 - \zeta^h) b \tag{74}$$